GRUMMAN

FINAL REPORT

PROGRAM MANAGEMENT AID

for

Redundancy Selection and Operational Guidelines

January 1972

CONTRACT NAS 10-7697

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REDUNDANCY SELECTION AND OPERATIONAL GUIDELINES

January 1972

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1.0 INTRODUCTION

The effective application of the knowledge and experience derived during the previous decade's space efforts, constitutes one of the major challenges and opportunities for the space programs envisioned for this decade, particularly in the areas of technical and management innovations. Significant cost reductions can be anticipated through the use of quantitative economic decision making aids. This report culminates six months of study effort by the Grumman Aerospace Corporation; under contract (NAS 10-7697) to NASA's John F. Kennedy Space Center, to develop an economic decision criterion for redundancy selection and operational guidelines.

The study was conducted in three phases as outlined below.

- Phase A - Update Redundancy Cost Criterion Nomograph previously developed by Grumman and provide documentation describing its use.

- Phase B - Update and provide documentation describing the "Launch Go/No Go" criterion (also previously developed by Grumman).

- Phase C - Development of an expanded cost criterion redundancy selector which integrates the two criteria mentioned above as well as other pertinent identified redundancy drivers.

2.0 SUMMARY

The shuttle program operational costs can be significantly reduced by judicious selection of redundancy for vehicle equipments and mechanisms during the DDT&E phase of the program. Under this contract, Grumman has developed management aids which enable the assessment of the economic consequences for various redundancy policies. These aids are applicable to many multimission spacecraft, aircraft or mechanisms as well as the shuttle spacecraft. The methodology employed is directly applicable even if the tools (nomographs and equations) are not, for mission peculiar cases.

The "integrated redundancy selection criterion" developed for estimating the economic consequences of various redundancy levels evolved from consideration of both the immediate economic consequences for design and procurement as well as the long term operational cost impacts for delays, reflown missions, checkout, failure replacement, repairs, etc. Evaluation of these identified cost impacts concluded that four were primary: operational delays, reflown missions due to aborts, procurement of equipment and vehicle growth to accommodate the added equipment. Thus, the integrated criterion developed under this contract considers only these four constituents, since the optimal redundancy level is relatively insensitive to the others.

Nomographs have been developed to enable "table top" sensitivity analysis to be performed. This tool gives good, clear answers but is not recommended for multiple configuration analysis with many equipments because of the complex iterative process employed. A more efficient computerized method of performing these multiple analyses is recommended.

2.0 SUMMARY

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Nomographs have been developed to enable "table top" sensitivity analysis to be performed. This tool gives good, clear answers but is not recommended for multiple configuration analysis with many equipments because of the complex iterative process employed. A more efficient computerized method of performing these multiple analyses is recommended.

During the operational phase of the shuttle program, decisions establishing whether to launch or repair subsequent to a redundant equipment malfunction on the launch pad must be rendered. The "GO/NO GO" launch criterion developed enables the economic consequences of these decisions to be assessed. This criterion is employed in the integrated redundancy selection criterion during the design phase and then used to reassess these preliminary decisions during the operational phase, since the operational data enables updating of initial data element estimates. Nomographs have also been developed for this criterion to aid in performing this analysis.

Grumman has been using portions of these tools in the Shuttle low cost avionics study in order to assess the economic consequences of redundancy for electronic equipment. The study concluded that a non-uniform redundancy policy was the most economical approach and if a uniform redundancy policy were instituted, FO/FS was the best level. These conclusions were presented to the NASA Avionics Study Team. In their final report of Avionic System Recommendations presented on November 29, 1971, NASA has changed their redundancy standard from FO^2/FS to a policy of selecting redundancy on a "cost effective" basis with analyses to be performed on an individual system basis.

The initial redundancy selection criterion used in the avionics study generally was found to suggest the same level of redundancy as the new integrated criterion. However, there are exceptions which occur when the operational delay costs combined with the pre-launch failure frequency are significant. For example, an additional Shuttle Inertial Measurement Unit which will eliminate delays is shown to produce an expected operational phase cost savings of one and three-quarter million dollars. In order to achieve the operational savings, an initial expenditure of about one and one-quarter

million dollars is estimated for procurement and vehicle growth. Thus, the net expected program cost savings is about one-half million dollars. This one example alone warrants the extra time and effort required to consider the integrated redundancy selection criterion over the simpler initial redundancy selection criterion which did not include the delay aspects.

In addition to the economic considerations, there are other aspects such as amount of mission degradation, state of the art of equipment, past program experience, safety and yearly program budget, which must be considered by management in order to render an equitable decision. Thus, the tools developed are meant solely as aids in expediting the decision process and not intended as the automatic decision maker.

3.0 APPROACHES TO REDUNDANCY SELECTION

Redundancy selection in the past was generally based on an allocated numerical probability level or allocated number of replicate units. These approaches were justifiable since the scarcity of data precluded the use of accurate quantitative analysis. A decade and more of space experience has provided sufficient data on failure rates of elements, costs of operation, and consequences of malfunctions to enable reasonably accurate economic assessments of various redundancy policies to be made.

The selection of redundancy to be included aboard the shuttle spacecraft will have an impact on the probability of launching on schedule, mission success, mission costs and total program costs. Figure 1 shows schematically the various mission outcomes. The ideal mission would consist of on time pre-launch operation, launch, payload delivery and landing safely. Of course, malfunctions will occur and the off nominal paths of delay, scrubs and aborts (shown shaded) in Figure 1 will result. By adding redundancy, the frequency of such off nominal missions will be reduced, but this requires an investment in equipment. Thus, the objective of this study is to find the proper balance between operational and initial investment costs which will minimize the overall expected program cost.

Since the shuttle program is a manned program, any approach which compromises the crew's safety is not amenable to this type of economic analysis. Therefore, the redundancy strategies which are considered are limited to those over and above a "fail safe" system. Figure 2 shows some of these alternatives. The "fail safe" unit consists of two units for mission operation purposes with mission abort if a single failure occurs. Thus the vehicle contains sufficient
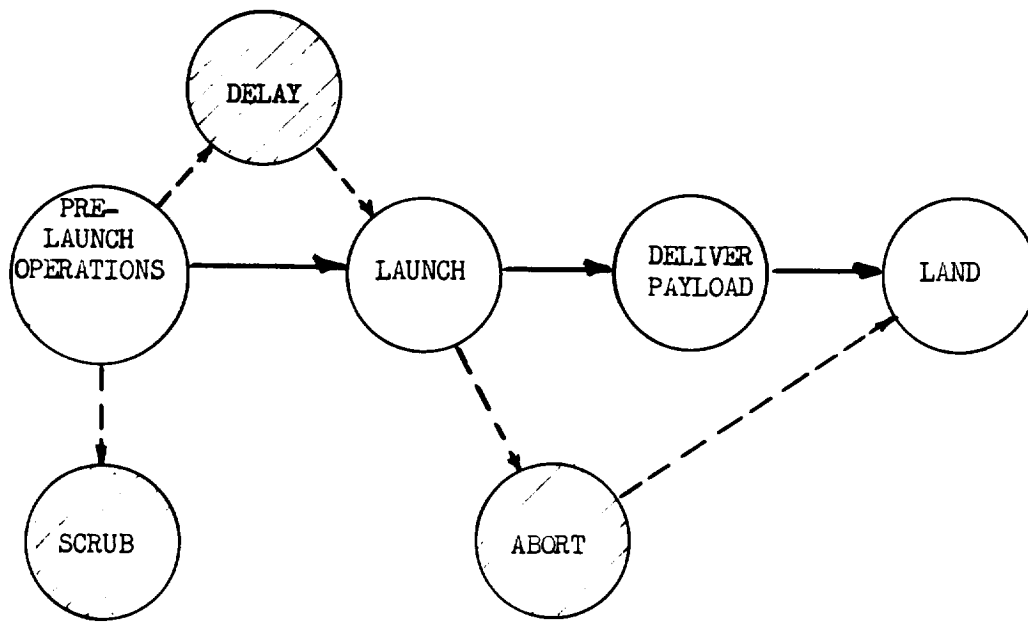
FIGURE 1 - <u>MISSION FLOWGRAPH</u>

equipment to get back to the base safely if no further malfunctions occur. The redundancy considered from an economic viewpoint is that over and above the "fail safe" system which will decrease the frequency of aborting the mission and is termed "fail operational" (FO).

The techniques employed in this economic analysis are generally termed cost-benefit analyses. Figure 3 depicts this process using a balance scale. On one side of the scale is placed the program cost benefits accrued from adding a redundant unit such as reduced aborted missions, reduced frequency of delays and fewer lost payload targets. On the other side of the balance scale are the costs incurred, such as procurement, vehicle growth, increased checkout time and additional failed equipment which must be repaired. If the analysis indicates that benefits outweigh the costs then the redundancy is added, whereas if the converse is true then the redundancy is not added.

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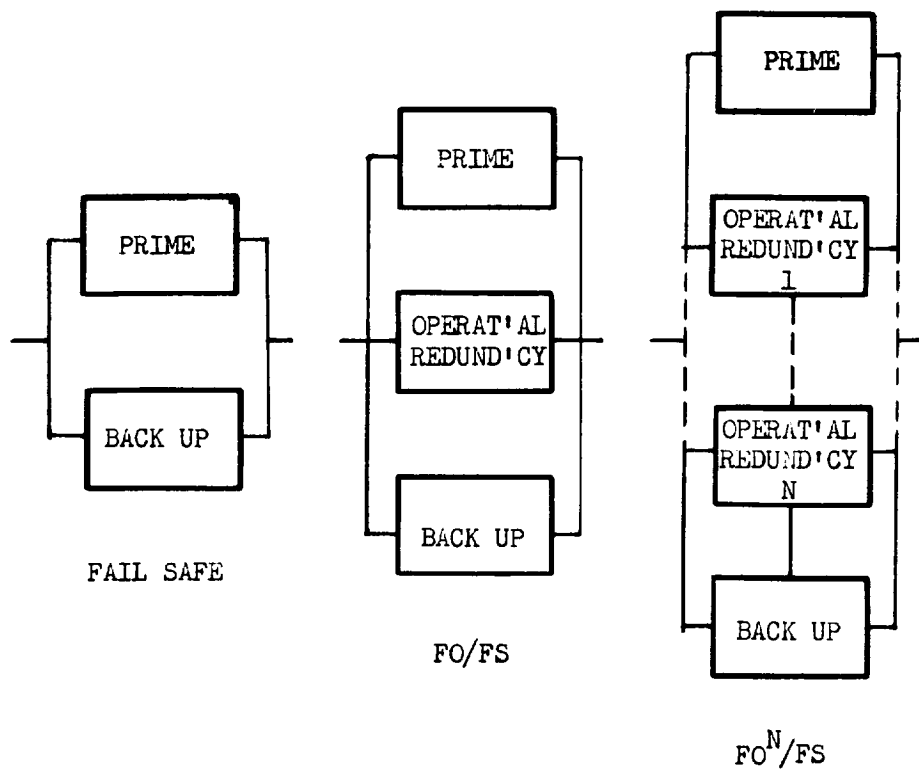
FIGURE 2 - <u>ALTERNATIVE REDUNDANCY STRATEGIES</u>

- Reduced Aborts
- Reduced Delays
- Reduced Scrubs
- Fewer Lost Targets

- Procurement
- Growth
- Incr. C/O
- Incr. Repairs
- Incr. Remove &
  Replace Actions
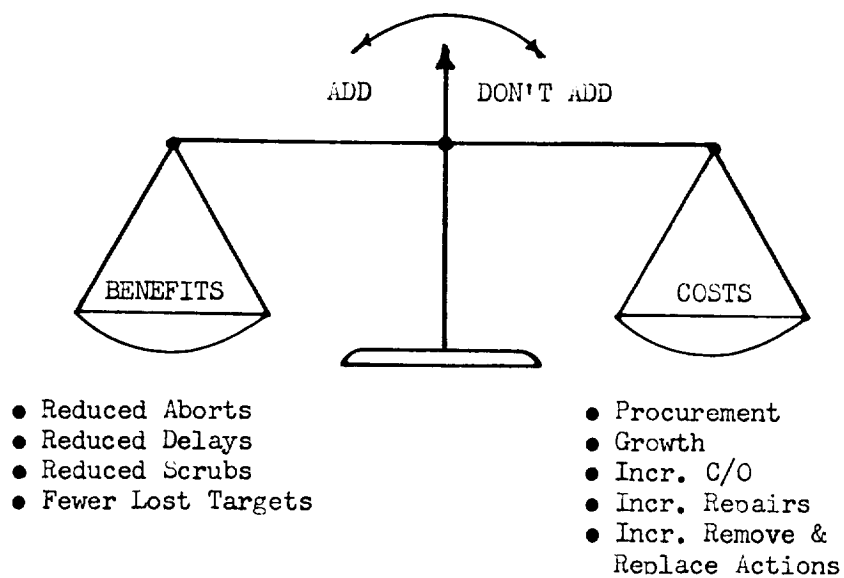
FIGURE 3 - <u>POTENTIAL BENEFITS AND COSTS DUE TO</u>
<u>INCREASING THE LEVEL OF REDUNDANCY</u>

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Figure 4

SHUTTLE OPERATIONAL REDUNDANCY SELECTOR

GRUMMAN
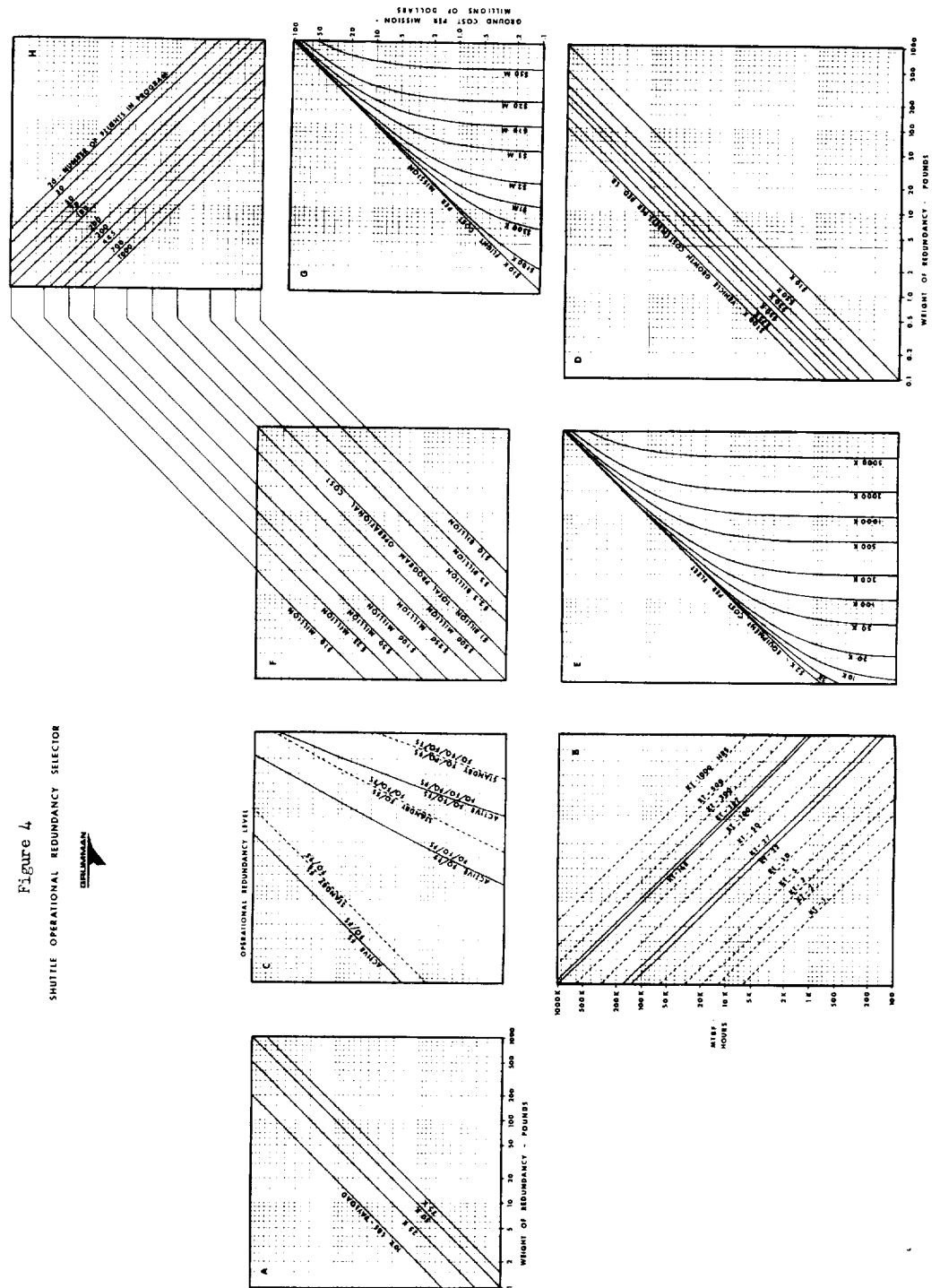
CPI = vehicle growth cost per pound of weight added to the vehicle

NF = number of flights planned during the program

GC = pre-launch ground operational costs per flight

FC = post launch mission operational costs

4.3 UPDATED INITIAL REDUNDANCY SELECTION NOMOGRAPH

The nomograph included here as Figure 4 is the updated version, wherein the ground operations costs were separated from the flight operations costs. Appendix I contains a description of the procedure for its use.

4.4 SENSITIVITY ANALYSIS

The nomograph was updated in order to assess the sensitivity of the redundancy level to ground operations cost. Therefore, several iterations for different sets of parameter values were made on the redundancy level to establish the relative sensitivity of the Ground cost. It was concluded that the ground cost was not the most sensitive nor the least sensitive but was in the middle. The sensitivity index was established by calculating the change in parameter value necessary to alter the redundancy one level in either direction and dividing it by the nominal parameter value.

$$\text{Sensitivity Index} = \frac{\text{Parameter} \left| \begin{array}{c} FO^{n+1} \\ FO^{n-1} \end{array} \right.}{\text{Parameter @ } FO^{n}}$$

Table 1 contains an example of the sensitivity analysis. The lower the sensitivity index, the smaller the parameter percentage change necessary to alter the recommended redundancy level, thus, the more sensitive the parameter. As is evident, the parameters cluster at three levels of sensitivity with the ground cost being in the middle and sixth out of nine overall.

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TABLE 1

SENSITIVITY ANALYSIS

	PARAMETER VALUE			SENSITIVITY INDEX
	NOMINAL (FO^2)	FO^3	FO	
STRESS FACTOR (KT) - HRS	27	55	15	1.5
MEAN TIME BETWEEN FAILURE (MTBF) - HRS	1000	450	2000	1.55
COST OF REDUNDANCY (CR) - $K	262	10	932	3.5
NUMBER OF VEHICLES (NV)	5	0	18	3.6
NUMBER OF FLIGHTS (NF)	445	3500	120	7.6
GROUND COST (GC) - 10^6	4	38	.45	9.4
COST PER POUND (CPI) - $K	40	--	1000	25.0
WEIGHT OF REDUNDANCY (WT) - LBS	3.5	--	90	25.7
FLIGHT COST (FC) - 10^6	1	35	--	35.0

4.5 APPLICATION OF INITIAL REDUNDANCY SELECTION CRITERION

The initial redundancy selection criterion was used during the shuttle low cost avionics study to assess the economic consequence of various redundancy policies. Figure 5 shows the results of this study. The expected program cost is seen to be lowest for the set of redundancies specified using this criterion, whereas, if a uniform redundancy policy was adhered to then FO/FS is the most economical level.

Cost = (Abort) + (Growth and Procurement over minimum FS system)
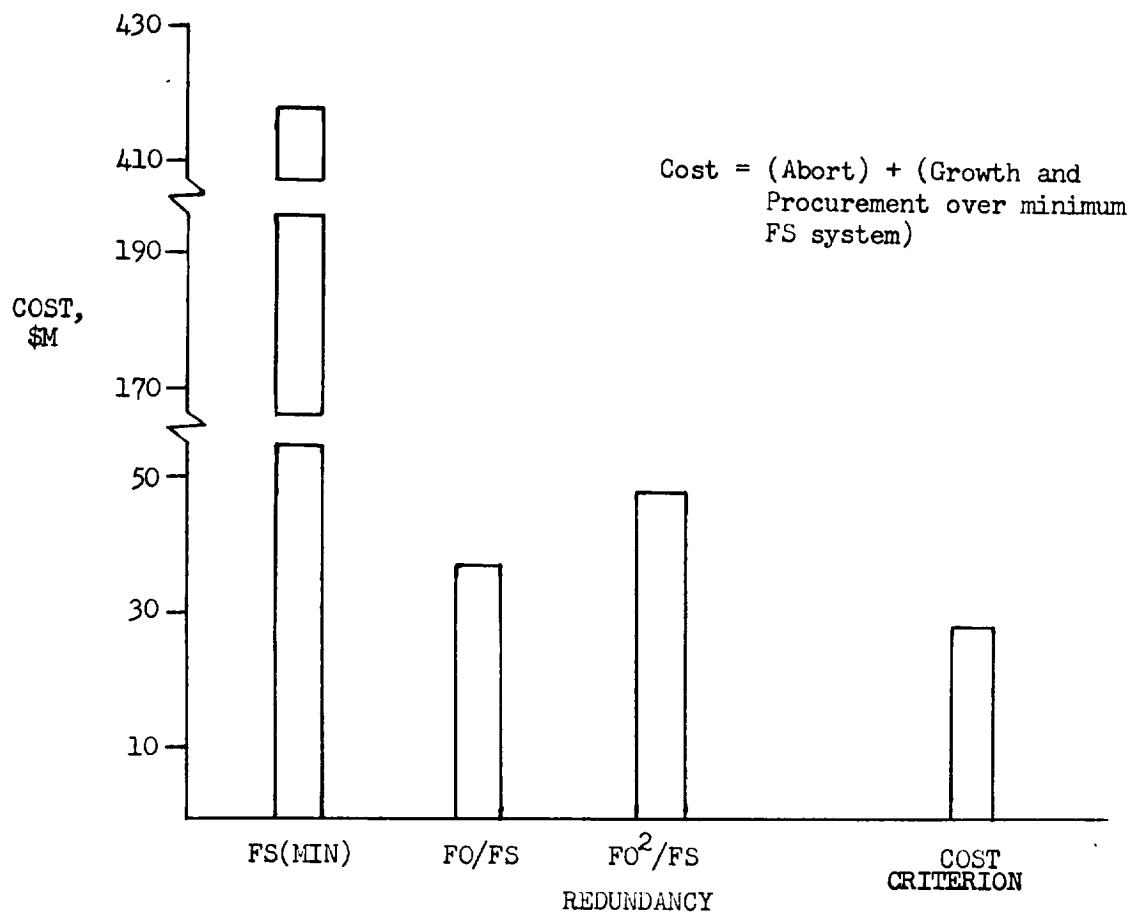
FIGURE 5 - <u>LOW COST AVIONICS STUDY REDUNDANCY LEVEL IMPACT ON PROGRAM COST</u>

Some of the estimated equipment parameter values are specified in Table 2 together with the level of redundancy specified by this criterion for each equipment. Additional information used in this study applicable to all equipment was a mission time of 168 hours, three vehicles in fleet, three hundred forty six flights, a cost per inert pound for vehicle growth of $16,500, and a combined mission ground and flight cost of $4.3 million.

In Section 6.9 of this report it will be shown that a more economical solution can be found for the inertial measurement unit, when the delay costs are considered. Generally the redundancy level specified under this criterion is consistent with that specified by the more complex integrated criterion to be discussed in Section 6.

TABLE 2

EQUIPMENT INCLUDED IN SHUTTLE LOW COST AVIONICS STUDY

EQUIPMENT PARAMETER VALUES

EQUIPMENT NAME	SUBSYSTEM	WEIGHT	COST	MTBF	RED. LEVEL
APU	EPS	200.0	200000	10000	FO/FS
Fuel Cell	EPS	115.0	200000	7500	
Inverter	EPS	40.0	3000	55000	
Trans & Rect A	EPS	20.0	3000	100000	
Trans & Rect B	EPS	10.0	3000	100000	
Regulator	EPS	15.0	10000	100000	
S Band XCVR	COMM	13.2	50000	10000	
Signal Processor	COMM	10.0	13000	30000	
Audio Center	COMM	5.0	7500	50000	
KU Band	COMM	32.0	23000	30000	FS
Mass Memory	DATA	10.0	15000	10000	FO^2/FS
Data Aquist Cont	DATA	15.0	19500	19000	FO/FS
Remote Acquist Unit	DATA	1.6	10500	72000	
Keyboard	D&C	15.0	20000	33000	
IMU	GN&C	57.0	118000	3500	FO^2/FS
Gen Computer	GN&C	75.0	100000	3400	FO^2/FS
Main Eng Cont	GN&C	3.5	1000	15000	FO/FS
ME TVC Elect	GN&C	7.8	12000	7500	
ACPS Cont Elect	GN&C	43.2	5000	26000	
Omes Cont & Thrtle	GN&C	10.0	14400	28000	
T-TCA	GN&C	5.7	17000	50000	
Rate Gyro	GN&C	3.5	51800	10000	
Accelerometer	GN&C	1.8	36000	66000	
Air Data Sensors	GN&C	.4	4000	70000	
Air Data Computer	GN&C	20.0	90000	30000	
Rot SS Cont	GN&C	5.5	10000	75000	
Speed Brake Cont	GN&C	7.0	10000	50000	
Aerosurf Trim Cont	GN&C	7.0	10000	50000	

TABLE 2   (Cont'd.)

EQUIPMENT INCLUDED IN SHUTTLE LOW COST AVIONICS STUDY

EQUIPMENT PARAMETER VALUES

EQUIPMENT NAME	SUBSYSTEM	WEIGHT	COST	MTBF	RED. LEVEL
Rudder Pedal Cont	GN&C	7.0	10000	50000	FO/FS
Fly By Wire FCS	GN&C	18.8	54000	5000	
Dir Man Backup FCS	GN&C	3.0	15000	20000	
Sec Actuator-Rud&SB	GN&C	20.0	13000	20000	
Pri Actuator-Red&SB	GN&C	100.0	13000	25000	
Aerospace Cont Elect	GN&C	3.3	8400	22000	
Sec Actuator-Elev	GN&C	20.0	13000	20000	
Pri-Actuator-Elev	GN&C	100.0	13000	25000	

5.0 PHASE B - UPDATE LAUNCH GO/NO GO CRITERION

5.1 PHASE B TASK

A criterion was developed, also in support of in-house Grumman Shuttle studies, to assess the economic consequences of launching subsequent to malfunction of one of several redundant units of equipment. This criterion was updated as part of this study. This task consisted of separating the flight and ground costs, development of a new nomograph and documentation describing its use.

5.2 LAUNCH "GO/NO GO" CRITERION

Subsequent to a flight vehicle equipment failure on the launch pad during the launch countdown sequence, a launch on schedule (GO) or a delay and repair (NO GO) decision must be rendered. This criterion weighs the economic consequences of each option to determine, in the long run, which approach will minimize the program cost. If the equipment configuration remaining after a failure is less than "fail safe" the vehicle should not be launched no matter how economical it is, since the next potential failure could cause injury to the crew. Therefore, the criterion developed weighs the economic implications for operational redundancy failures only.

The additional costs incurred by delaying the launch consists of the product of the time duration for the delay and the cost per hour for the personnel either actively involved in repairing the malfunction or the idle personnel waiting for the countdown sequence to commence again. The additional risk assumed if the repair is not made will result in more aborted missions. Thus, a launch without repair (GO) is recommended if the following inequality holds.

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$$\Delta R < \frac{HD \times CH}{GC + FC} \tag{2}$$

where ΔR = increase in abort probability due to the elimination of one level

of redundancy

HD = hours of delay to make repair

CH = cost per hour of delay

GC = pre-launch ground operational costs per flight

FC = post launch mission operational cost per flight

5.3 LAUNCH "GO/NO GO" CRITERION NOMOGRAPH

The nomograph for performing this analysis is included as Figure 6, wherein the ground operations costs are separated from the flight operations costs. Appendix II contains a description of this nomograph and a procedure for its use.

5.4 ESTIMATE OF DELAY COSTS

An estimate of the delay costs for holds at various points in the pre-launch operations sequence were made.

Three cases were considered and are outlined below:

- Case I - Failure occurs prior to fuel loading with no built-in hold periods remaining until scheduled launch

- Case II - Failure occurs after fueling which commences approximately 4 to 6 hours prior to launch during the early development flight program. The delay is anticipated to be less than 18 hours to make the repair

- Case III - Same as Case II but the repair time is more than 18 hours. This necessitates dumping of the fuel back into storage and refueling

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Figure 6

SHUTTLE GO/NO-GO SELECTOR

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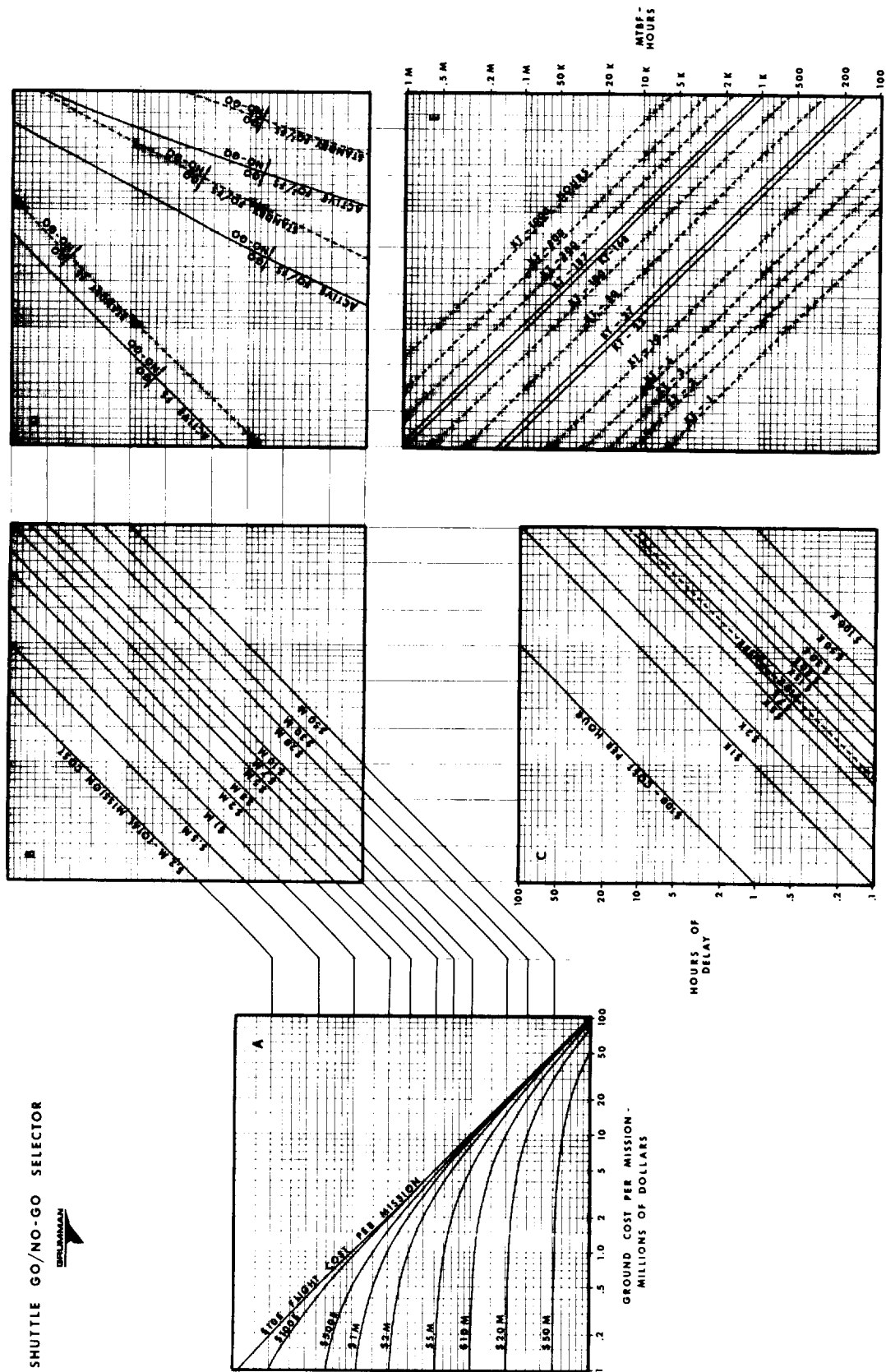
after repair. The launch vehicle fuel boil-off rate is such that all the fuel anticipated to be stored will be consumed in an eighteen hour hold period

The costs associated with delay in Case I are composed of the idle hours for the anticipated crew of 91 launch technicians and mission control personnel. The sum of these personnel costs is estimated at $1,000 per hour.

In Case II, the estimated fuel and oxidizer boil-off rates are 4%/hr and 2 1/4%/hr. respectively. Boil-off replenishment will cost about $7,000 per hour. This estimate was based on 2.5 million pounds of liquid oxygen and 400,000 pounds of liquid hydrogen being onboard with an assumed cost of $30 per ton for oxygen and $800 per ton for hydrogen. In addition to the fuel costs, the same 91 people are required, contributing another $1,000 per hour. Thus, the total is $8,000 per hour.

For Case III, in addition to the $1,000 per hour personnel costs, there is a fixed cost of approximately $250,000 incurred in dumping the fuel from the launch vehicle back into storage and then refilling the tanks.

5.5 APPLICATION OF LAUNCH GO/NO GO CRITERION

The bulk of the electronic equipment analyzed during the shuttle low cost avionics study are located in the crew compartment. These equipments have been estimated to require two to four hours to isolate the fault, remove and replace the malfunctioned unit and verify the integrity of the repaired system, subsequent to a malfunction on the launch pad. Those equipments which are located outside the crew compartment are not as accessible and the time to isolate and replace these units is estimated to be about twice that of the ones located inside. Thus, for purposes of this application a delay cost of $32,000 (4 hours @ $8,000 per hour) and $64,000 (8 hours @ $8,000 per hour) was

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assumed for equipment located inside and outside the crew compartment respec-
tively. The mission cost, ground plus flight, was assumed to be $4.3 million.
Table 3 identifies the various equipments and the most economical approach
for each subsequent to a malfunction, therefore, launch or delay and repair.
Of the thirty-six equipments analyzed, only four would be most economically
treated by repairing the malfunction on the launch pad. These four are the
S-band transceiver, KU band, data acquisition unit and the inertial
measurement unit.

In Section 6.9 of this report, it will be shown that by applying the
integrated criterion, which considers the delay cost and frequency thereof,
an additional level of redundancy is recommended for the inertial measurement
unit, over that recommended using the initial selection criterion of Section 4
of this report.

TABLE 3

EQUIPMENT INCLUDED IN SHUTTLE LOW COST AVIONICS STUDY

ESTIMATED PARAMETER VALUES FOR GO/NO GO LAUNCH CRITERION

EQUIPMENT NAME	SUBSYSTEM	MTBF	DELAY COST	LAUNCH DECISION
APU	EPS	10000	$64,000	GO
Fuel Cell	EPS	7500	64,000	
Inverter	EPS	55000	32,000	
Trans & Rect A	EPS	10000	64,000	
Trans & Rect B	EPS	10000	32,000	
Regulator	EPS	10000	32,000	
S Band XCVR	COMM	10000	32,000	NO GO
Signal Processor	COMM	30000	32,000	GO
Audio Center	COMM	50000	32,000	GO
KU Band	COMM	30000	32,000	NO GO
Mass Memory	DATA	10000	32,000	GO
Data Acquist Cont	DATA	19000	32,000	NO GO
Remote Aquist Unit	DATA	72000	64,000	GO
Keyboard	D&C	33000	32,000	GO
IMU	GN&C	3500	32,000	NO GO
G&N Computer	GN&C	3400	32,000	GO
Main Eng Cont	GN&C	15000	64,000	
ME TVC Elect	GN&C	7500	64,000	
ACPS Cont Elect	GN&C	26000	64,000	
Omes Cont & Thrtle	GN&C	28000	64,000	
T-TCA	GN&C	50000	64,000	
Rate Gyro	GN&C	10000	64,000	
Accelerometer	GN&C	66000	64,000	
Air Data Sensors	GN&C	70000	64,000	
Air Data Computer	GN&C	30000	32,000	
Rot SS Cont	GN&C	75000	32,000	
Speed Brake Cont	GN&C	50000	32,000	
Aerosurf Trim Cont	GN&C	50000	32,000	

TABLE 3 (Cont'd.)

EQUIPMENT INCLUDED IN SHUTTLE LOW COST AVIONICS STUDY

ESTIMATED PARAMETER VALUES FOR GO/NO GO LAUNCH CRITERION

EQUIPMENT NAME	SUBSYSTEM	MTBF	DELAY COST	LAUNCH DECISION
Rudder Pedal Cont	GN&C	50000	$32,000	GO
Fly By Wire FCS	GN&C	5000	32,000	
Dir Man Backup FCS	GN&C	20000	32,000	
Sec Actuator-Rud&SB	GN&C	20000	64,000	
Pri Actuator-Rud&SB	GN&C	25000	64,000	
Aerospace Cont Elect	GN&C	20000	64,000	
Sec Actuator-Elev	GN&C	20000	64,000	
Pri-Actuator-Elev	GN&C	25000	64,000	

6.0 PHASE C - INTEGRATED REDUNDANCY SELECTION CRITERION

6.1 APPROACH

The objective of this portion of the study was to incorporate the delay costs and any other significant cost drivers into the redundancy selection criterion. This initially required an enumeration of all program related costs which vary with the levels of redundancy. Then an estimate of the nominal magnitude and range of values that each cost element could assume was made in order to determine the relative magnitude of each and identify the primary ones. After synthesizing the evaluation criterion a nomograph was built to enable quick sensitivity analyses to be performed. This section of the report documents the procedure for applying this criterion as well as its development.

6.2 CANDIDATE COST CONSTITUENTS

Alteration of the amount of redundancy impacts both the initial investment costs as well as the operational costs. The initial investment costs are one time fixed costs whereas the operational costs occur periodically due to changes in performance characteristics and are statistical in nature. The costs have been divided into two groups and are delineated below together with definition of each.

- Initial Investment Costs
    - Procurement - The cost to purchase and install one additional unit of equipment in each shuttle flight vehicle
    - Growth - The cost for additional structure, propulsive capability, power, etc., which must be designed into the vehicle configuration, to accommodate the added weight of redundancy.

- **Recurring Operational Costs**
  - **Aborts** - Each mission that is terminated prematurely can be considered as either a wasted mission or one which must be reflown in order to deliver the payload. Thus, the ground operations required to turn around the vehicle and the mission operation and launch costs must be spent again
  - **Checkout** - Each redundant unit must be monitored during prelaunch operations in order to ascertain vehicle status. This required additional time and manpower and additional equipment or in some cases new equipment and procedures
  - **Delay** - Every time a malfunction occurs, which according to the "GO/NO GO" criterion is more economical to repair, a schedule delay is incurred. Direct maintenance time and manpower for replacing malfunctioned equipment as well as idle personnel time must be accounted for here
  - **Remove and Replace** - With additional redundant units, more potential failures exist, consequently there will be more remove and replace actions. These post flight or ground operations costs include manpower and equipment to perform these maintenance actions
  - **Repairs** - Each equipment that malfunctions is either thrown away or repaired. Assuming fault detection capability is built-in or achievable at some lower level off line, most equipment will require a component replacement and be rotated back into inventory

6.3 PRIMARY COST CONSTITUENTS

To establish the relative magnitude of each cost element contained in Section 6.2, a survey of typical electronics equipment envisioned for the

low cost shuttle avionics system was made. Table 2 (Ref. Section 4.5), as noted previously, is a listing of these equipments together with an estimate of each unit's cost, weight and mean time between failure. From this list of diverse equipment, an estimate of the typical value of each parameter was made as follows; an MTBF of 25,000 hours, weight of redundancy of 20 pounds and cost of redundancy of 15,000 dollars. In estimating the nominal value for each criterion cost constituent these values were used with the exception of the $15,000 unit cost which was escalated to $50,000 to reflect the cost of new equipment instead of modified off-the-shelf equipment envisioned as part of the low cost avionics program. Additional data used in estimating the magnitude and range of each cost constituent were:

Delay Cost - $32,000

KT Ground - 50

KT Flight - 200

Number of Vehicles - 4

Number of Flights - 500

Growth Cost per Pound - $32,000

The cost constituent's magnitude was estimated for each of the most likely levels of redundancy, FO/FS and FO^2/FS, using the following equations:

Abort Cost = Probability of Abort X Number of Flights X Reflown Mission Cost

Growth Cost = Cost per Pound X Weight of Redundant Unit

Procurement Cost = Cost per Unit X Number of Vehicles

Delay Cost = Cost per Delay X Number of Missions X Probability of a Delay

Checkout Cost = Number Hours for Checkout X Number of Personnel X Manhour Cost

Repair Cost = Cost per Repair X Number of Flights X Probability of a
Failure in Flight or On Ground

Remove and Replace Cost = Cost per Manhour X Number of Manhours per
Action X Probability of Failure in Flight
or On Ground X Number of Flights

Since the cost benefit analysis technique makes use of the marginal value
the difference between the two levels of redundancy is the quantity of inter-
est. Table 4 contains these differential constituent cost estimates using the
previously mentioned nominal values for the typical equipment. Also, the
range was estimated for each due to variation in the mean time between failure,
number of flights and procurement cost. It can be readily noted that the
ordinal ranking of each cost constituent stays constant but the relative mag-
nitude of each cost varies as the parameter estimates are varied.

The ordinal ranking and magnitude of each cost element was:

	Magnitude
ABORTS	10^6
GROWTH	5×10^5
PROCUREMENT	5×10^5
DELAYS	10^5
REPAIRS	5×10^3
CHECKOUT	5×10^3
REMOVE/REPLACE	10^2

Figure 7 shows isometrically the cost constituent's relative magnitude
(median value). Identified also is an estimate of the range for each cost
element. As is evident from the table and figure, the seven cost constituents
magnitude can be divided into two groups; the aborts, growth, procurement and
delay costs in one range and then, more than an order of magnitude removed,
the checkout, repairs and remove and replace costs. Since it would take many

TABLE 4

COST CONSTITUENTS NOMINAL MAGNITUDE AND VARIATION

	PROCUREMENT COST 50,000				PROCUREMENT COST 100,000				AVERAGE
	MTBF 25,000		MTBF 10,000		MTBF 25,000		MTBF 10,000		
NO. FL	100	500	100	500	100	500	100	500	
DELAY	*2×10^4	9×10^4	5×10^4	2×10^5	2×10^4	9×10^4	5×10^4	2×10^4	9×10^5
CHECKOUT	2×10^3	10^4	2×10^3	10^4	2×10^3	10^4	2×10^3	10^4	6×10^3
REMOVE & REPLACE	20	10^2	40	2×10^2	40	10^2	40	2×10^2	90
PROCUREMENT	2×10^5			↑	4×10^5		↑	↑	3×10^5
REPAIRS	10^3	5×10^3	4×10^3	2×10^4	10^3	5×10^3	4×10^3	2×10^4	5×10^3
GROWTH	4×10^5	6×10^5	4×10^5	6×10^5	4×10^5	6×10^5	4×10^5	6×10^5	5×10^5
ABORTS	2×10^5	10^6	5×10^5	2×10^6	2×10^5	10^6	5×10^5	2×10^6	9×10^5

Program Cost Constituents

*Each Cell Entry is in Units of Dollars

ABORTS   GROWTH   PROCUREMENT   DELAY   CHECKOUT   REPAIRS   REMOVE & REPLACE

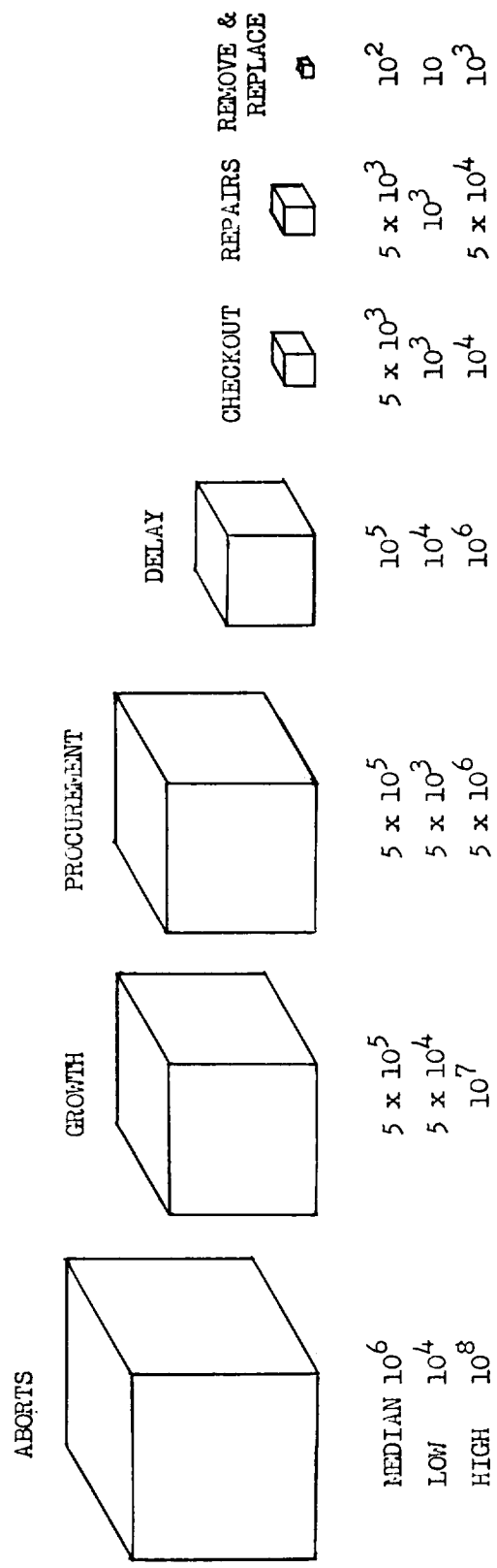|	ABORTS	GROWTH	PROCUREMENT	DELAY	CHECKOUT	REPAIRS	REMOVE & REPLACE
MEDIAN	10^6	5×10^5	5×10^5	10^5	5×10^3	5×10^3	10^2
LOW	10^4	5×10^4	5×10^3	10^4	10^3	10^3	10
HIGH	10^8	10^7	5×10^6	10^6	10^4	5×10^4	10^3

FIGURE 7 - RELATIVE MAGNITUDE AND RANGE OF PROGRAM COST ELEMENTS
(FO/FS → FO^2/FS REDUNDANCY)

of these lower group cost constituents to equal one of the higher group, these three cost constituents can be ignored with little loss in accuracy.

6.4 FIGURE OF MERIT

The measure or figure of merit for each redundancy level is the expected program cost. Therefore, the objective of this criterion is to select the redundancy level which minimizes the expected program cost. Figure 8 is a pictorial representation of this. The total program cost consists of a baseline cost which is the aggregate of development, testing, procurement, design and operational costs for the nominal flight schedule for all candidate redundancy levels. To these baseline costs are added the significant operational costs which vary with the level of redundancy; the abort or reflown mission costs and the delay costs.

Addition of a redundant unit (FO/FS) brings some operational costs down and some up. Potential abort costs decrease since the system reliability increases, whereas the delay costs increase since there are more units which could potentially fail. Additional units must be procured for each flight vehicle and the checkout, repair and replacement costs increase because of more units and more failures. These trends continue in this pattern, as additional redundancy is added, as long as malfunctions are repaired as they occur on the launch pad. Once it becomes uneconomical to repair malfunctions, as determined by using the "GO/NO GO" criterion explained in Section 5 and Appendix II, then the pattern changes. For illustrative purposes, Figure 8 assumes that the FO^3/FS configuration is launched with a failure, whereas, the FO^2/FS configuration is not launched with a failure. As illustrated in the figure, the abort cost decreases with addition of FO^3/FS unit and the procurement, growth, checkout, repair and remove/replace costs increase as
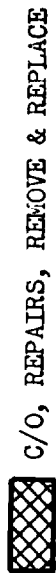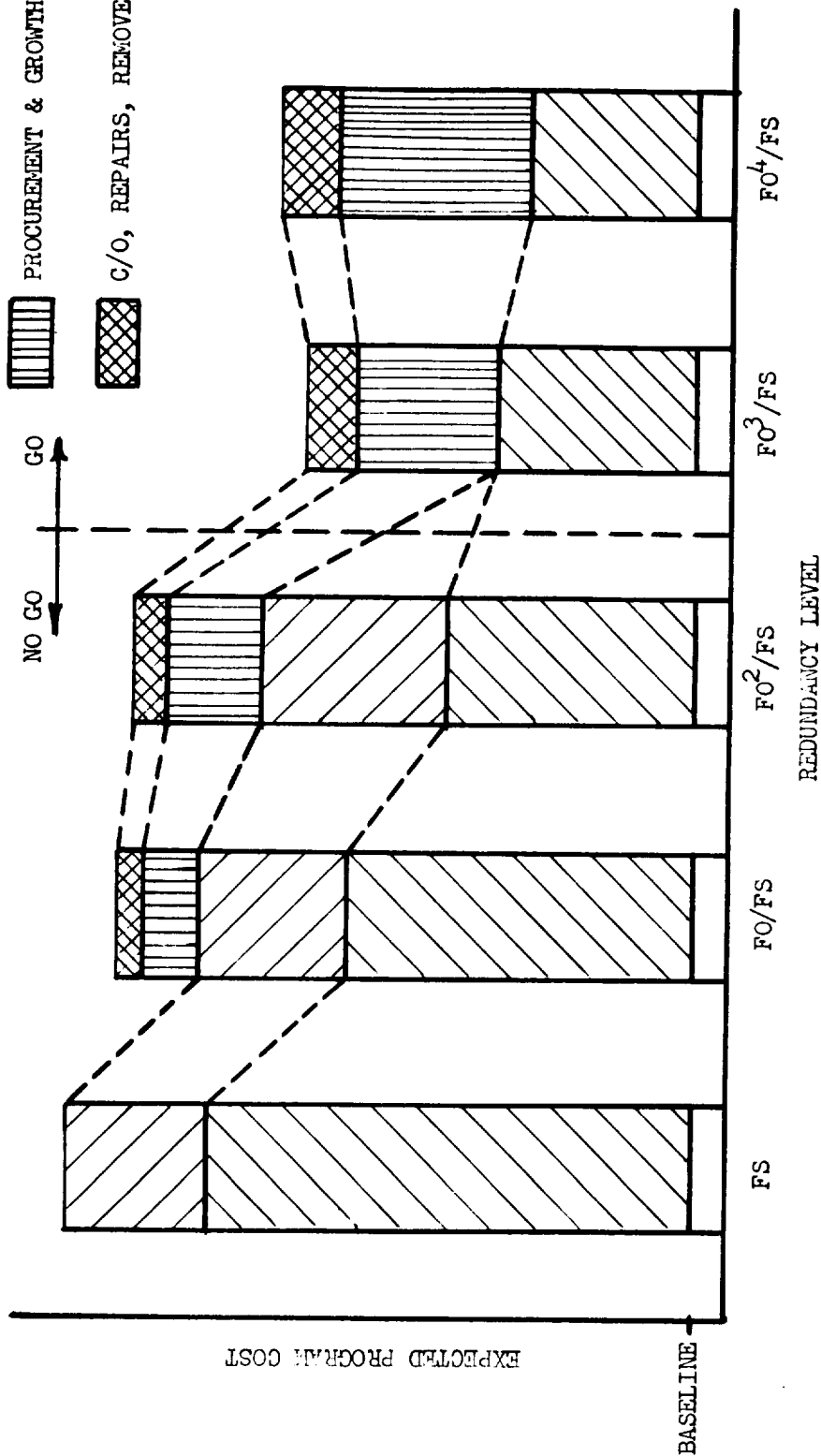
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FIGURE 8 - PROGRAM COST VARIATION WITH REDUNDANCY LEVEL

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before. However, the delay costs, instead of increasing, drop to zero since a pre-launch failure is not repaired at that time. By evaluating the aggregate differential or marginal cost, as each additional redundant unit is added, and noticing whether it is negative or positive, a judgement as to whether the total program cost is increasing or decreasing can be rendered. The procedure for applying this criterion then is to determine when the program cost starts increasing. In practical application this approach will ordinarily give the correct or at least a good answer. In theory, however, it is possible for the program cost to have two local minimum areas and the solution obtained by using the outlined procedure may not be the global minimum but reasonably close to it.

6.5 CRITERION

Since the marginal redundancy benefits and cost constituents vary with the launch "GO/NO GO" decision three unique criterion were developed in order to evaluate each specific case. The mathematical development of the equations is contained in Appendix III and a summary of each follows.

6.5.1 "GO → GO" Redundancy Criterion

If the "GO/NO GO" launch criterion suggests launching without repair (GO) for each of two adjacent redundancy levels, then the Program cost differential is attributed to procurement, growth and abort costs. The delay costs are zero in both cases since malfunctions are not economical to repair prior to launch.

The procurement cost is the total cost of one additional unit for all of the vehicles in the fleet. The growth cost is the propulsive fuel, bigger engines, structure, etc., added to each flight vehicle to accommodate the redundant unit. The abort costs are reduced since, usually, the vehicle is launched with a greater number of operable redundant units. Since malfunctions are not repaired the vehicle can be launched in either of two states: all up

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or one failed. Thus the abort probability is the weighted average of the various launch states.

Redundancy is added under this criterion if the expected program cost with N units of equipment is lower than N-1 units, where N is any arbitrary level of redundancy. The details of the derivation of this criterion equation is Case I of Appendix III. The following inequality must be satisfied in order to make the Nth unit economical to add.

$$\Delta R_N > \left(\frac{C_R N_V + W_R C_P}{(GC + FC) N_F} - P_{N-1} (F) \cdot \Delta R_{N-1} \right) \frac{1}{1 - P_N(F)} \qquad (3)$$

where: C_R = Procurement cost of a single unit

N_V = Number of vehicles in the fleet

W_R = Weight of redundant unit

C_P = Cost per lb for vehicle growth

GC = Ground turnaround costs

FC = Flight operations costs

N_F = Number of flights

ΔR_N = Change in probability of mission success by adding N^{th} unit

$P_N(F)$ = Probability of failure during pre-launch checkout with N units of equipment

6.5.2 "NO GO → GO" Redundancy Criterion

This redundancy criterion applies when with N units of an equipment designed into the spacecraft, the "GO/NO GO" launch criterion advocates launching with a failure; whereas if only N-1 units of equipment are designed into the vehicle, the "GO/NO GO" launch criterion advocates not launching with a failure. Therefore, the addition of the Nth unit of equipment eliminates all potential delays due to a malfunction in one of these N equipments.

Since the additional redundant unit provides a higher probability of mission success, the benefit of eliminating delay costs, as well as reducing the frequency of aborting, must be weighed against the costs of procuring the units for the fleet of vehicles as well as the vehicle growth cost due to the additional weight being carried. The reduction in flight abort frequency is not as high as might be thought at first glance. With N-1 units and a "NO GO" launch decision subsequent to failure the vehicle will always be launched with N-1 units operative. However, with N units and a "GO" launch decision subsequent to failure sometimes the vehicle is launched with N units operative and sometimes N-1 units operative. When the "GO" configuration of N units is launched with one unit failed, the abort probability is the same as the "NO GO" configuration of N-1 units. Thus, the flight abort frequency is only reduced for those percentage of missions where the "GO" configuration is launched with N units operative.

An additional unit of redundancy is added under this criterion if the expected program cost is lower with N units than with N-1 units. Appendix III, Case II contains the details and derivation of the following inequality which must be satisfied in order for it to be economical to add the Nth unit of redundancy under this criterion.

$$\Delta R_N > \frac{1}{1 - P_N(F)} \left(\frac{C_R N_V + W_R C_P}{(GC + FC) N_F} - \frac{C_D P_{N-1}(F)}{(GC + FC)} \right) \tag{4}$$

where: C_R = Procurement cost of a single unit

N_V = Number of vehicles in the fleet

W_R = Weight of redundant unit

C_P = Cost per lb for vehicle growth

GC = Ground turnaround cost

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FC = Flight operations cost

N_F = Number of flights

C_D = Cost of delay to repair a malfunction

$P_N(F)$ = Probability of failure during pre-launch operations with N units of equipment

ΔR_N = Change in probability of mission success by addition of the Nth unit

6.5.3 "NO GO → NO GO" Redundancy Criterion

Additional redundancy may not always eliminate delay costs. When the "GO/NO GO" criterion advocates repairing malfunctions that occur on the launch pad whether there are N-1 or N units of equipment installed in the vehicle, then the marginal benefits and costs are described by the "NO GO → NO GO" criterion.

The benefits due to addition of the Nth unit result solely from the reduction in abort frequency. Costs incurred are three-fold; procurement, vehicle growth and additional delays. Additional delays are encountered because there are more units which can potentially fail. However, this is only true with an active, operating form of redundancy. If the redundancy is passive (standby) then the additional unit does not increase the delay frequency. Thus for this case the criterion becomes identical to the initial redundancy selection criterion described in Section 4 of this report.

An additional unit is added under this criterion if the expected program cost is lower with N units than with N-1 units. If the following inequality is satisfied then it is economical to add the Nth unit of equipment.

$$\Delta R_N > \frac{C_R N_V + W_R C_P}{(GC + FC) N_F} + \frac{C_D \Delta D_n}{(GC + FC)} \qquad (5)$$

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where: C_R = Procurement cost of a single unit

N_V = Number of vehicles in the fleet

W_R = Weight of the redundant unit

C_P = Cost per lb for vehicle growth

GC = Ground Turnaround cost

FC = Flight operations cost

N_F = Number of flights in program

C_D = Cost of delay to repair a malfunction

ΔR_N = Change in probability of mission success by addition of the Nth unit

$\Delta D_N = P_N(F) - P_{N-1}(F)$ = change in probability of a delay on the launch pad due to addition of the Nth unit

$P_N(F)$ = Probability of failure during pre-launch operations with N units of equipment

6.5.4 Other Redundancy Criterion

It is sometimes possible, as mentioned in Section 6.4, that the solution using any of criterion 6.5.1 through 6.5.3 yields a local optimum and not a global optimum. In other words, the appropriate criterion, when applied, may suggest that it is not economical to add the next level of redundancy, whereas, if several additional redundant units had been considered it would have been economical. As an illustration of this, see Figure 9. This hypothetical case assumes that the expected program cost drops by adding the first redundant unit (FO/FS). The second redundant unit (FO^2/FS) has a higher expected program cost than the FO/FS configuration. Each of these cases would have been evaluated using the "NO GO $\rightarrow$ NO GO" redundancy criterion of Section 6.5.3. Application of the criterion would have suggested FO/FS as being the best level of redundancy from an economic viewpoint.

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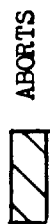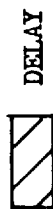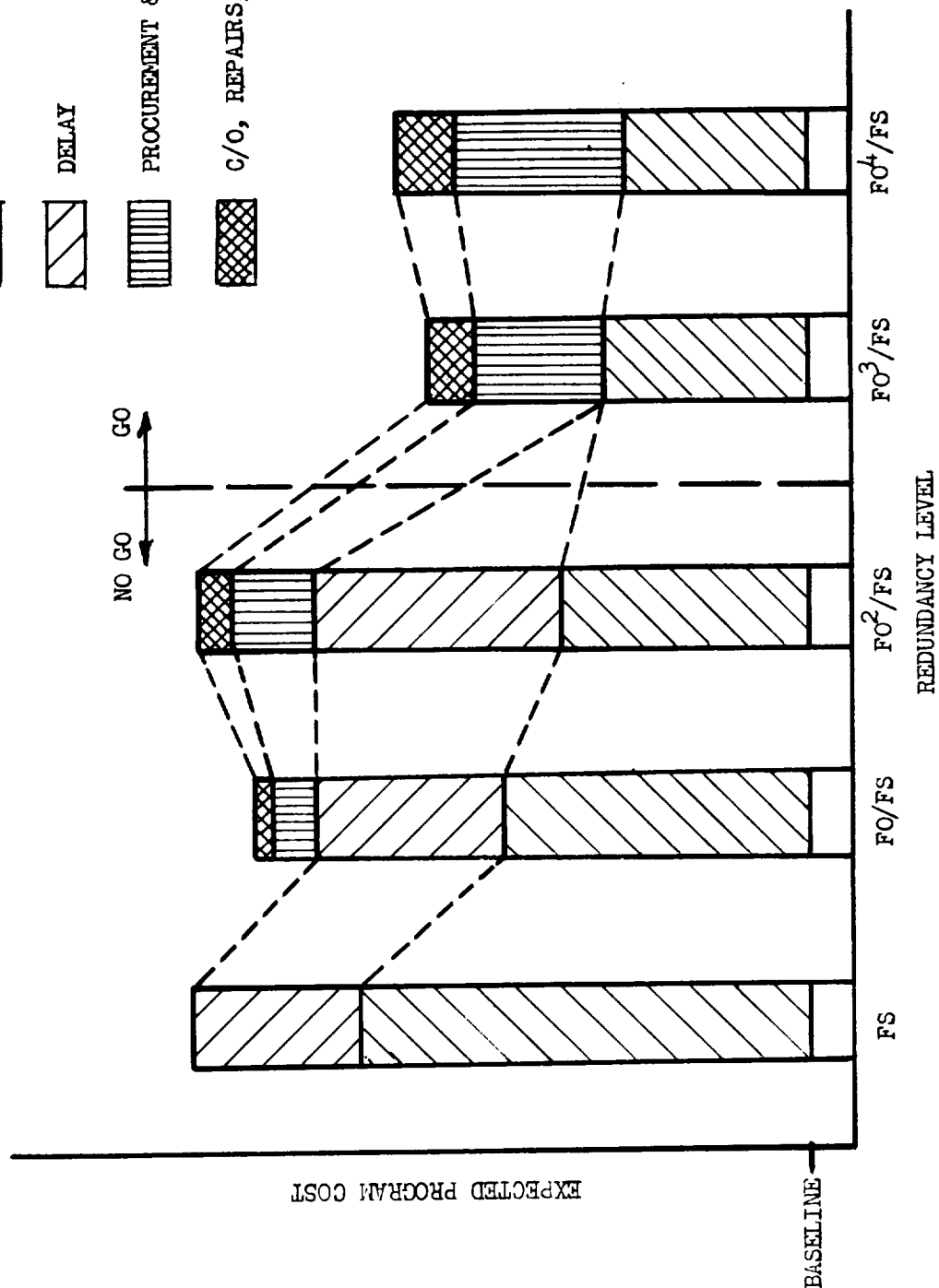
FIGURE 9 - PROGRAM COST OSCILLATION WITH REDUNDANCY LEVEL

When the redundant unit is added which enables the vehicle to be launched with failures then a considerable savings may be realized. This is assumed to occur when the FO^3/FS unit is added in our hypothetical example. Elimination of this considerable delay cost may result in a lower expected program cost for the FO^3/FS configuration than the FO/FS configuration. The inequality which must be satisfied for these several levels of redundancy to be added is:

$$\Delta R_{N+I} > \frac{(I) \cdot (C_R N_V + W_R C_P) - \left\{ (GC + FC) \ N_F \cdot \sum_{x=n+1}^{n+I-1} \Delta R_X \right\} - P_N (F) \cdot C_D \cdot N_F}{(GC + FC) \cdot (1 - P_{N+I} (F))} \quad (6)$$

where: N = Last unit that was economical to add

$N+I$ = First unit that enables vehicle to be launched with an equipment malfunctioned

C_R = Procurement cost of a single unit

N_V = Number of vehicles in the fleet

W_R = Weight of the redundant unit

C_P = Cost per pound for vehicle growth

GC = Ground turnaround cost

FC = Flight operations cost

N_F = Number of flights in the program

ΔR_N = Change in probability of mission success due to addition of the Nth unit

$P_N(F)$ = Probability of failure during pre-launch operations with N equipments

C_D = Cost of a delay to repair a malfunction

This criterion, in practice, will be applied very infrequently, however, it is theoretically possible.

Other theoretical cases exist which occur even less frequently and thus since the purpose of this contract is to develop tools for practical applications they have been pursued no further.

6.6 EVALUATION OF STATISTICAL ASPECTS OF CRITERION

Each of the decision criterion consists of both deterministic and probabililistic decision elements. Those deterministic elements, such as procurement, ground and flight costs, are the ones which we can predict accurately each time because the underlying causes are identifiable. Those which vary from trial to trial and are unpredictable except from a probability viewpoint, such as number of failures, number of delays and number of aborted flights, require statistical treatment. This section discusses those decision elements which are statistical in nature and their method of evaluation.

There are two types of redundancy principally used in most spacecraft designs; standby (non-operative) and active (operative). It is possible and practical to have a combination of active and standby redundant units in one system but this will not be treated.

The standby form of redundancy is typically applied where short down times can be tolerated. A sensing device detects the failure of the operating unit and through some logic circuitry one of the redundant units is switched on while the malfunctioning unit is switched off. The failure rate of the offline non-operative redundant unit is assumed to be zero. This type of process where the arrival rate of failed units is constant over time is described as a Poisson process. The probability of any number of arrivals (n) in some time period given that each individual unit also has a constant

failure rate with time (exponential failure time density function) is described mathematically as

$$P_n(T) = e^{-KT/MTBF} \frac{(KT/MTBF)^n}{n!} \qquad (7)$$

where: $P_n(T)$ = Probability of exactly n arrivals in time T

K = Environmental stress factor

T = Time period of observation

MTBF = Mean time between failures

The active or operating form of redundancy is typically applied where no down time can be tolerated or the switching logic and circuitry necessitates a very complex mechanism. In this type of process the failure rate of each unit is assumed constant with time but the rate of units failing, which is the sum of the failure rates for all operating units, is not constant with time since as one unit fails there are less units operating. It is obvious that with this form of redundancy the number of units failing in any given time period will be higher than with the standby. Thus, a group of n equipments will have a lower reliability with the active form of redundancy than the standby.

The active redundancy is represented by a binominal process and is described mathematically, if each individual unit has an exponential failure time density function, as:

$$P_{X/n}(T) = \binom{n}{x} (1-e^{KT/MTBF})^x (e^{-KT/MTBF})^{n-x} \qquad (8)$$

where: $P_{X/n}(T)$ = Probability of exactly x failures in time t with n units at time zero operative

$\binom{n}{x}$ = Combinational of n things taken x at a time = $\frac{n!}{x!\,(n-x)!}$

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K = Environmental stress factor

T = Time period of observation

MTBF = Mean time between failure

The three terms used in the redundancy criterion which require statistical evaluation are the change in abort probability (ΔR_N) due to addition of the Nth unit of equipment, the change in delay probability (ΔD_N) due to addition of the Nth unit of equipment and the probability of a failure $[P_N(F)]$ on the launch pad with N units of equipment. Table 5 summarizes the equations for each statistical term and for various redundancy levels. The derivation of each follows:

ΔR_N - Change in Abort Probability

Under the groundrules stated in the introduction a shuttle mission is aborted if the next potential failure could cause injury to the crew. Thus with N units onboard, the mission is aborted if N-1 failures occur, whereas if only N-1 units are onboard then the mission is aborted when N-2 failures occur.

Prob. of aborting = 1 - Prob. of not aborting

$$P_N(A) = 1 - P_N(\bar{A})$$

$$P_N(A) = 1 - \sum_{x=0}^{N-2} P_x(t)$$

$$P_{N-1}(A) = 1 - \sum_{x=0}^{N-3} P_x(t)$$

$$\Delta R_N = P_{N-1}(A) - P_N(A)$$

$$= 1 - \sum_{x=0}^{N-3} P_{x/N-1}(t) - 1 + \sum_{x=0}^{N-2} P_{x/N}(t)$$

$$= \sum_{x=0}^{N-2} P_{x/N-1}(t) - \sum_{x=0}^{n-3} P_{x/N}(t)$$

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TABLE 5

EQUATIONS FOR CALCULATING STATISTICAL TERMS OF CRITERION

REDUNDANCY LEVEL	TYPE OF RED.	ΔR_N	ΔD_N	$P_N(F)$
FO/FS	STBY	$(KT/MTBF) \cdot P_1$	0	$1-P_1$
	ACT.	$-e^{-2KT/MTBF} + \sum\limits_{x=0}^{1} \binom{3}{x}(1-P_1)^x\, P_1^{\,3-x}$	$(P_1)^2 \cdot (1-P_1)$	$1-P_1^{\,3}$
FO^2/FS	STBY	$\dfrac{(KT/MTBF)^2}{2!} \cdot P_1$	0	$1-P_1$
	ACT.	$\sum\limits_{x=0}^{2} \binom{4}{x}(1-P_1)^x\, P_1^{\,4-x} - \sum\limits_{x=0}^{1} \binom{3}{x}(1-P_1)^x\, P_1^{\,3-x}$	$P_1^{\,3} \cdot (1-P_1)$	$1-P_1^{\,4}$
FO^3/FS	STBY	$\dfrac{(KT/MTBF)^3}{3!} \cdot P_1$	0	$1-P_1$
	ACT.	$\sum\limits_{x=0}^{3} \binom{5}{x}(1-P_1)^x\, P_1^{\,5-x} - \sum\limits_{x=0}^{2} \binom{4}{x}(1-P_1)^x\, P_1^{\,4-x}$	$P_1^{\,4}(1-P_1)$	$1-P_1^{\,5}$

$$P_1 = e^{-KT/MTBF}$$

Substituting the appropriate expressions for standby and active redundancy the following results.

Standby Redundancy

$$\Delta R_N = \sum_{x=0}^{N-2} e^{-KT/MTBF} \cdot \frac{(KT/MTBF)^x}{X!} - \sum_{x=0}^{n-3} e^{-KT/MTBF} \cdot \frac{(KT/MTBF)^x}{X!} \qquad (9)$$

Active Redundancy

$$\Delta R_N = \sum_{x=0}^{N-2} \binom{N}{x} (1 - e^{-KT/MTBF})^x (e^{-KT/MTBF})^{N-x}$$

$$- \sum_{x=0}^{N-3} \binom{N-1}{x} (1 - e^{-KT/MTBF})^x (e^{-KT/MTBF})^{N-x-1} \qquad (10)$$

where: N = Total number of units of equipment designed into vehicle

ΔR_N = Change in abort probability due to addition of the Nth unit of equipment

KT = Sum of products of environmental stress factor and operating time over all mission phases

$MTBF$ = Mean time between failure for a single unit.

ΔD_N - Change in Delay Probability

As additional redundant units are added the frequency of delays on the launch pad change because there are more potential units to fail. However, this is true only if the redundant units are operating. Since standby redundant units are not operating they do not increase the delay probability.

Standby Redundancy

$$\Delta D_N = 0$$

Active Redundancy

Probability of delay = Prob. of failure on launch pad

$$P_N(D) = P_N(F)$$

$$\Delta D_N = P_N(F) - P_{N-1}(F)$$

$$P_N(F) = 1 - \text{Prob. of no failure}$$

$$= 1 - \left(e^{-KT/MTBF}\right)^N \qquad (11)$$

$$\Delta D_N = 1 - \left(e^{-KT/MTBF}\right)^N - \left(1 - \left(e^{-KT/MTBF}\right)^{N-1}\right)$$

$$= \left(e^{-KT/MTBF}\right)^{N-1} \cdot \left(1 - e^{-KT/MTBF}\right)$$

where: $P_n(F)$ = Probability of failure with n units of equipment

$P_n(D)$ = Probability of delay with n units of equipment

KT = Sum of products of environmental stress factor (k) and operating time (t) over all phases of pre-launch operations

MTBF = Mean time between failure of a single unit

The individual equipment failure density function is assumed to be exponential.

$P_n(F)$ - Probability of Failure on the Launch Pad

The probability of failure calculation is identical to the probability of delay since a failure causes a delay. The probability of failure with the standby form of redundancy is constant regardless of the quantity of redundancy since only one unit is operating. Addition of the active form of redundancy causes an increase in the failure probability.

Standby Redundancy

$$P_N(F) = 1 - e^{-KT/MTBF} \qquad (12)$$

Active Redundancy

$$P_N (F) = 1 - \text{Prob. of no failure}$$

$$P_N (F) = 1 - (e^{-KT/MTBF})^N \tag{13}$$

where: N = Number of units of equipment installed in vehicle

KT = Sum of products of environmental stress factor (k)

and operating time (t) for all pre-launch operations

$MTBF$ = Mean time between failure for single unit

It is assumed that the failure density function for a single unit is exponential.

6.7 ANALYSIS PROCEDURE

As is obvious from the many criteria that must be employed, the procedure for establishing the most economical level of redundancy for an equipment is quite complex.

Figure 10 contains a flow graph of the redundancy selection process. It is an iterative process employing the "GO/NO GO" launch criterion as well as the several redundancy criterion of Section 6.5.

Each of the steps of the procedure are outlined below and correlated with Figure 10 through the numbers located in each block of the flow graph.

- Block 1 - Start the analysis process here by specifying the minimal quantity of good units required to launch the vehicle. Therefore, the quantity for which a malfunction on the launch pad would necessitate a delay because there would be less than a "fail safe" system and the next potential failure could cause injury to the crew.

- Block 2 - Using the "GO/NO GO" launch criterion described in Section 5.0 decide whether one additional unit of redundancy would obviate delays. If it does then go to Block 4. If not, then go to Block 3.
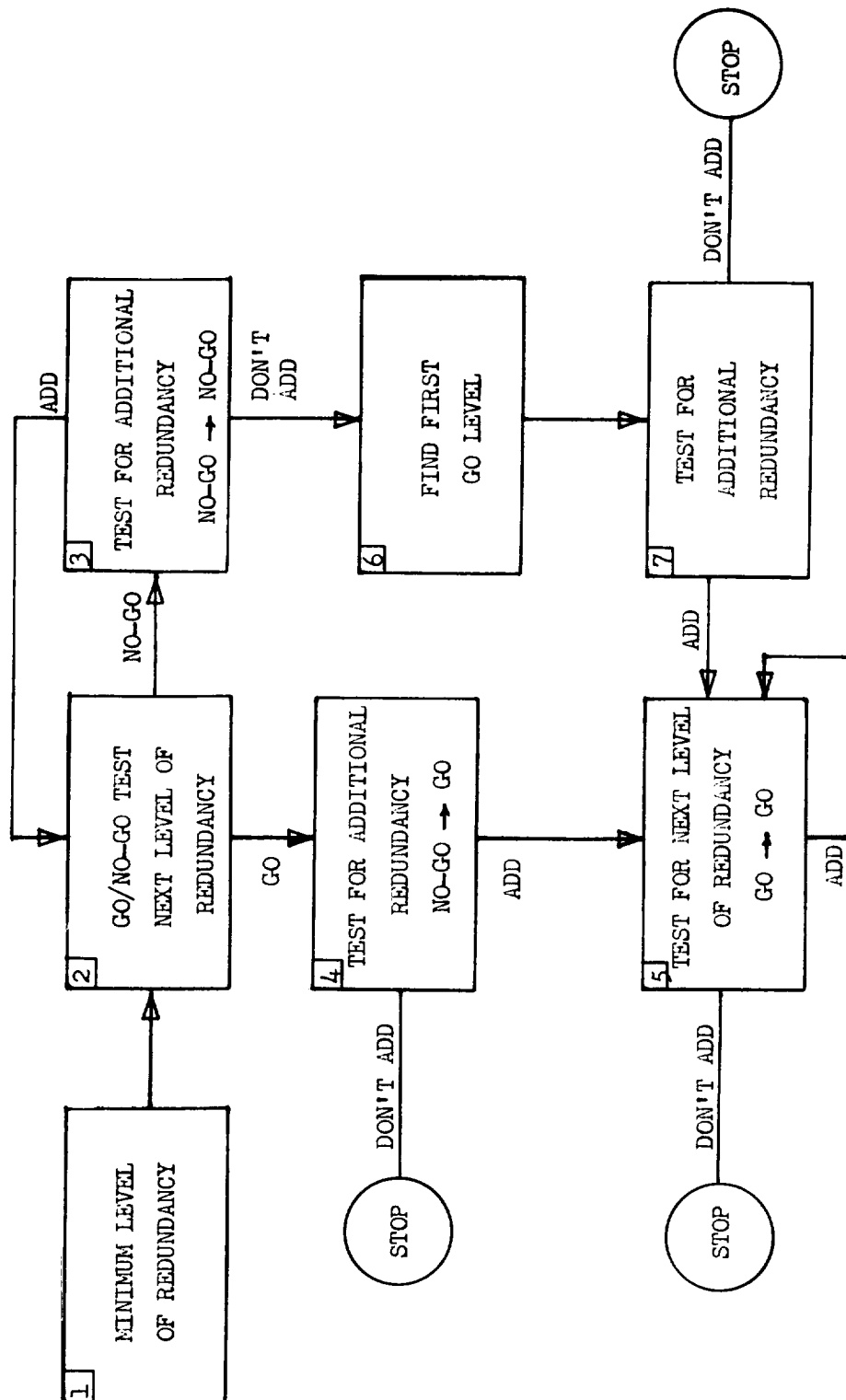
FIGURE 10 - REDUNDANCY SELECTION FLOW GRAPH

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- Block 3 - Using the "NO GO → NO GO" redundancy criterion of Section 6.5.3 evaluate the economics of adding this additional unit. If it is not economical to add it, then go to Block 6. If it is economical then go to Block 2 again.

- Block 6 - This block is entered when the analysis process would stop prior to considering the potentially large cost savings due to elimination of delay costs. Therefore, using the "GO/NO GO" launch criterion establish the first "GO" level of redundancy. Then proceed to Block 7.

- Block 7 - Using equations of Section 6.5.4 test to see if the total program cost is reduced when the several redundant units are added. If it is not economical to add these units then don't consider any further levels of redundancy. If it is economical then proceed to Block 5.

- Block 4 - This block is entered when the "GO/NO GO" launch criterion identifies that it is not economical to launch with a failure prior to addition of this redundant level but the additional redundancy makes it economical to launch with a failure. The "NO GO → GO" redundancy criterion of Section 6.5.2 is applied. If it is worthwhile to add the redundancy then Block 5 is entered. If it is not worthwhile then the evaluative process is terminated.

- Block 5 - When this block is entered the previous redundancy level and the new redundancy level under consideration both result in "GO" decisions when a malfunction occurs on the launch pad. Thus, the "GO → GO" redundancy criterion of Section 6.5.1 is applied to establish whether it is economical to add this additional redundancy. If it is not

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economical then the evaluative process is terminated. If it is

worthwhile then the criterion is applied again for the next level

of redundancy. This continues until an additional level of

redundancy is identified which is not economical to add.

6.8 NOMOGRAPHS

In order to enable quick "table top" tradeoffs and sensitivity analyses,

nomographs were developed for each individual criterion. Additionally, a

composite nomograph was generated on which, by using selected charts, each of

the criterion can be evaluated. These charts, besides performing the mathe-

matics pictorially, lend visibility to the procedure and credibility to the

assumptions.

The nomographs, together with the procedure for its use and a discussion

of the function of each chart composing it, are included in this section.

6.8.1 Function of Each Nomograph Chart

Many of the charts are repeated from nomograph to nomograph. The com-

posite nomograph, Figure 11, since it contains all charts, will be used as a

reference in describing the function of each chart. Each chart is coded

with a letter and all identical charts on other nomographs have the same

letters.

- Chart A - This chart is the decision making chart and is a plot of

    operational redundancy levels for equipment in either active or

    standby redundancy. The dashed lines for standby or non-operative

    redundancy were plotted from the Poisson equation. The solid lines

    for active or operating redundancy were plotted from the binominal

    equation.

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Figure 11

INTEGRATED REDUNDANCY SELECTION CRITERIA
COMPOSITE NOMOGRAPH
(GO→GO, NO-GO→GO, NO-GO→NO-GO)

GRUMMAN

$1-P_N(F)$
PROBABILITY OF NO PRE-LAUNCH FAILURE
WITH N UNITS OF EQUIPMENT
(FROM COMPUTATION NOMOGRAPH-CHART Q)

GO-GO CRITERION: ΔR_{N-1}
DECREASE IN ABORT PROBABILITY
DUE TO ADDITION OF N-1th UNIT
(FROM CHART A)
OR
NO-GO/GO CRITERION: $C_D/\theta C + FC$
(FROM GO/NO-GO SELECTOR-CHART Q)

FOLDOUT FRAME

GROUND COST PER MISSION - MILLIONS OF DOLLARS

WEIGHT OF REDUNDANCY - POUNDS

0.1 0.2 0.5 1.0 2 5 10 20 50 100 200 500 1000

ΔD N
CHANGE IN DELAY PROBABILITY

.001 .002 .004 .01 .02 .04 0.1 0.2

.02
.008
.004
.002
.0008
.0004
.0002
.00008
.00004
.00002

$C_D/\partial C + FC$
(FROM GO/NO-GO SELECTOR-CHART Q)

.001 .002 .004 .01 .02 .04 .1 .2 .4 1.0

$P_{N-1}(P)$
PROBABILITY OF PRE-LAUNCH FAILURE
WITH N-1 UNITS OF EQUIPMENT
(FROM COMPUTATION NOMOGRAPH-CHART N)

FOLDOUT FRAME 3

By entering on the abscissa with the sum of the products of the environmental stress factor and operating time for all mission phases divided by the MTBF and on the ordinate with the right hand side of the appropriate criterion inequality, the optimum level of redundancy can be established. The zone in which the abscissa and ordinate values intersect specifies the most economical level of redundancy.
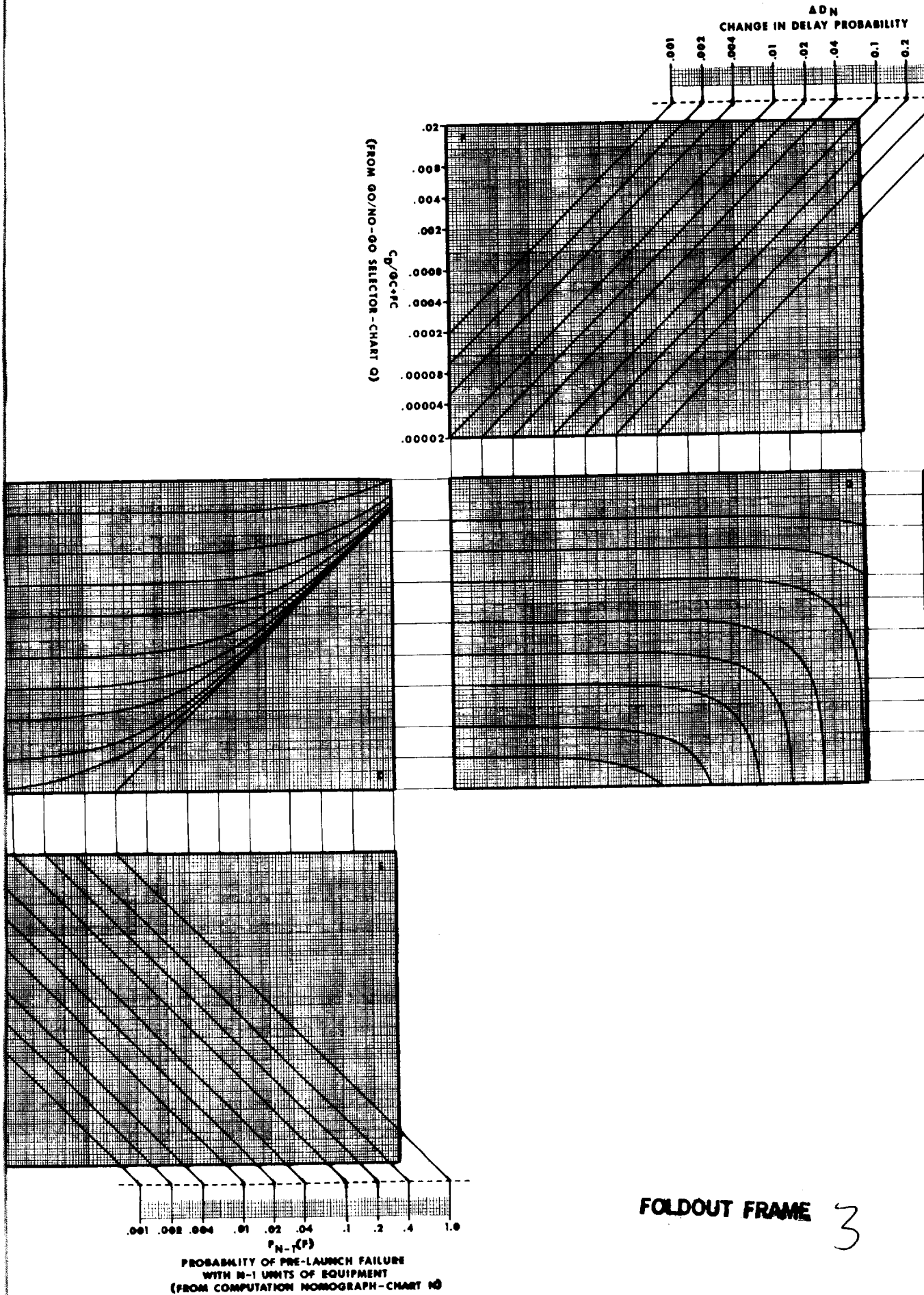
- Chart B - The function of this chart is to establish the value KT/MTBF which is the variable needed to establish ΔR for each level of redundancy in Chart A. Thus, the MTBF (mean time between failure) for a single unit of equipment specified on the ordinate is divided into the composite KT factors represented by the family of diagonal lines. This factor, KT, is the sum of the products of the time (T) that the equipment is in operation during each flight phase and the appropriate K factor which is dependent upon the environmental stress to which the equipment is subjected.

| Phase of Flight		Time of Operation (T) in Hours	Stress Factor (K)	Mission Duty Cycle Factor (KT)
Launch	Boost	0.1	10	1
	Glide	4.0	1	4
Oribtal Operation		160	1	160
Deorbit	Re-Entry	1.75	10	17.5
	Glide	2.0	1	2
	Landing	0.25	10	2.5
Launch & De-Orbit				27
Launch, Orbital Oper., & De-Orbit				187

- Chart C - In this chart the probability of no failure on the launch pad $(1-P_N(F))$, represented by the abscissa, is divided into the value coming from Chart D represented by the diagonal lines. This is used in both the "GO → GO" and "NO GO → GO" redundancy criterions. The diagonals represent a portion of the right hand side of the inequalities and the resultant of this division is the complete right hand side of the decision inequality.

- Chart D - This chart performs a subtraction function, subtracting the abscissa value from the ordinate value. It is used in both the "GO → GO" and "NO GO → GO" redundancy criterion.

- Chart E - The function of this chart is to multiply the value on the ordinate (either ΔR_{N-1} or $C_D/GC + FC$) by the probability of a malfunction on the launch pad $(P_{N-1}(F))$ assuming one less unit than presently under consideration were installed in the spacecraft. $P_{N-1}(F)$ is represented by the family of diagonal lines.

- Chart F - The function of this chart is to multiply the change in delay probability (ΔD_N) due to the nth unit of equipment and represented by a diagonal line, by the ordinate value representing the cost of delay divided by the single mission costs $(C_D/GC + FC)$.

- Chart G - The function of this chart is to add the outputs from Chart H (ordinate) and Chart F (abscissa) resulting in the right hand side of the "NO GO → NO GO" redundancy criterion inequality.

- Chart H - The function of this chart is to divide the total program flight and ground costs (diagonals) into the vehicle growth and equipment procurement costs (abscissa). This forms a portion of the right hand side of the inequality for all three criteria.

- Chart I - The function of this chart is to add the vehicle growth cost (ordinate) to the equipment cost for the fleet (family of curved lines). This is used in all three criterion.

- Chart J - The function of this chart is to multiply the single mission cost (abscissa) by the number of flights in program (diagonals) to obtain the total program cost. This is used for all criterion.

- Chart K - The function of this chart is to add the ground cost per mission (ordinate) to the flight cost per mission (family of curved lines) to obtain the single mission cost, which is used in all three criterion.

- Chart L - The function of this chart is to multiply the weight of a single unit (abscissa) by the vehicle growth cost per pound added to the vehicle (diagonals), obtaining the total vehicle growth cost over the planned flight program.

The following charts appear on the computation nomograph, Figure 12. These charts are used to perform preliminary statistical calculations prior to entering the individual criterion or composite nomographs.

- Chart B - This chart is identical to that on the other nomographs but its function is to establish the KT/MTBF factor for ground operations rather than flight operations.

- Chart M - The function of this chart is to determine the change in delay probability (ΔD_N) due to addition of a redundant unit. This is accomplished by subtracting the probability of failure with N units, $P_N(F)$, entered on the ordinate from the probability of failure with N-1 units entered on the abscissa. The curved family of lines in the chart specify this difference.

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Figure 12

INTEGRATED REDUNDANCY SELECTION CRITERIA
COMPUTATION NOMOGRAPH
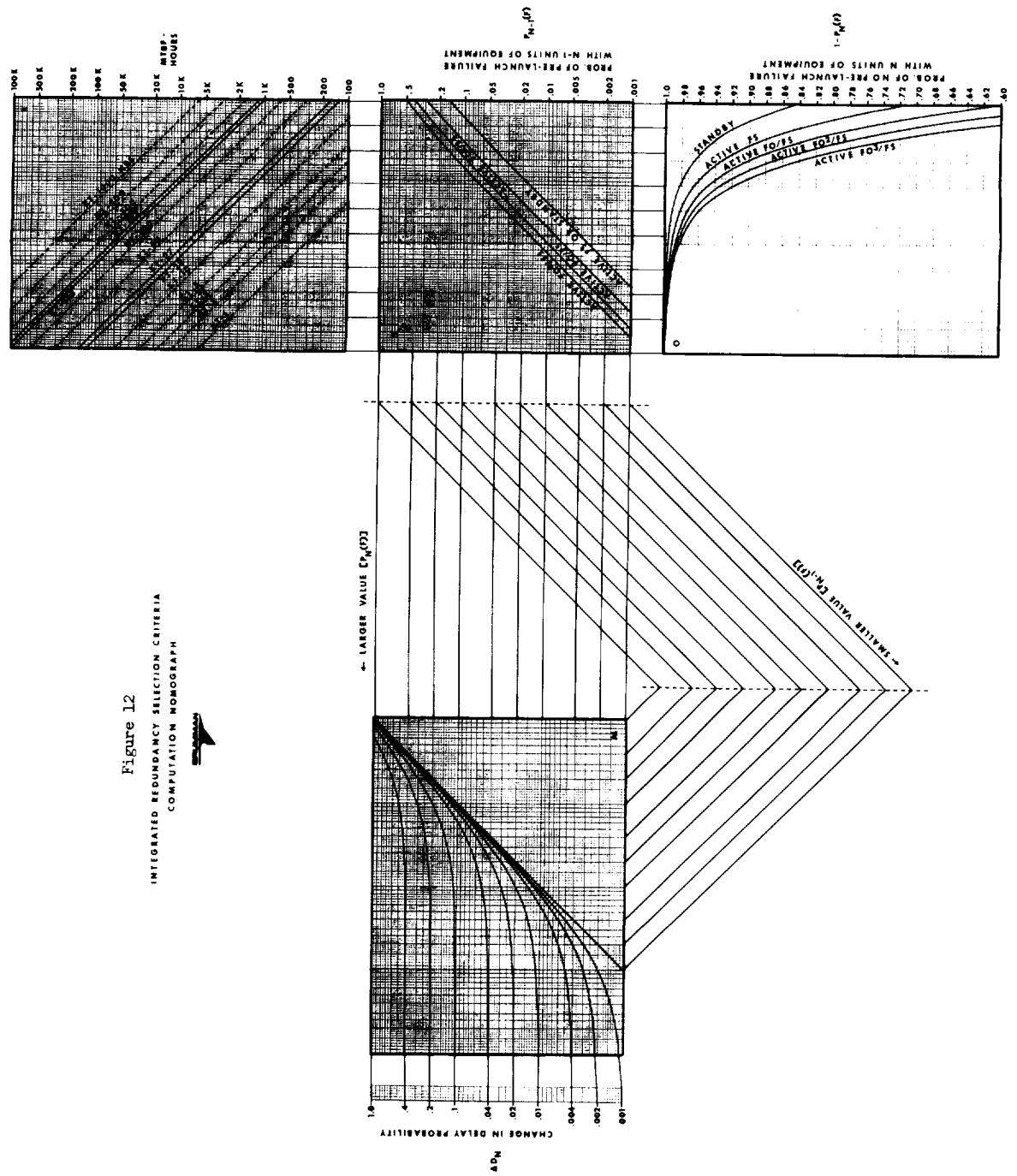
- **Chart N** - The function of this chart is to convert KT/MTBF factor (abscissa) into a probability of failure on the launch pad (ordinate) for the appropriate level of redundancy (diagonal). The output of this chart $(P_{N-1}(F))$ is used in the "GO → GO" and "NO GO → GO" redundancy criterion.

- **Chart O** - The function of this chart is to convert the pre-launch KT/MTBF factor (abscissa) into the probability of no malfunction $(1-P_N(F))$ on the launch pad, assuming N units of equipment were installed in the vehicle.

The updated "GO/NO-GO" launch criterion nomograph contained in Appendix II of this report has been modified slightly for use with the integrated criterion. The function of each chart contained in this nomograph, Figure 13, is as follows.

- **Chart B** - This chart has the same function as that for the composite

- **Chart K** - The function of this chart is identical to the same lettered chart on the composite nomograph. However, it has been rotated ninety degrees so the ordinate and abscissa are reversed.

- **Chart P** - The function of this chart is to decide whether it is more economical to launch with a failure or delay the launch and repair the malfunction. The intersection of the ordinate value and abscissa value obtained from Charts Q and B when compared to the level of redundancy prior to an equipment malfunction accomplishes this.

- **Chart Q** - The function of this chart is to divide the delay cost (abscissa) by the single mission cost (diagonal), obtaining the right hand side of the "GO/NO GO" launch criterion inequality. Also this value is used as an input to both the "NO GO → GO" and "NO GO → NO GO" redundancy selection criterion nomographs.

Figure 13

INTEGRATED REDUNDANCY SELECTION CRITERIA

SHUTTLE GO/NO-GO SELECTOR

GRUMMAN

- Chart R - The function of this chart is to multiply the hours of delay (ordinate) by the cost per hour of delay (diagonals) and obtain the total delay cost (C_D) which is the abscissa.

Table 6 is a summary of each of the chart functions using the symbols specified in the appropriate portion of Section 6.5.
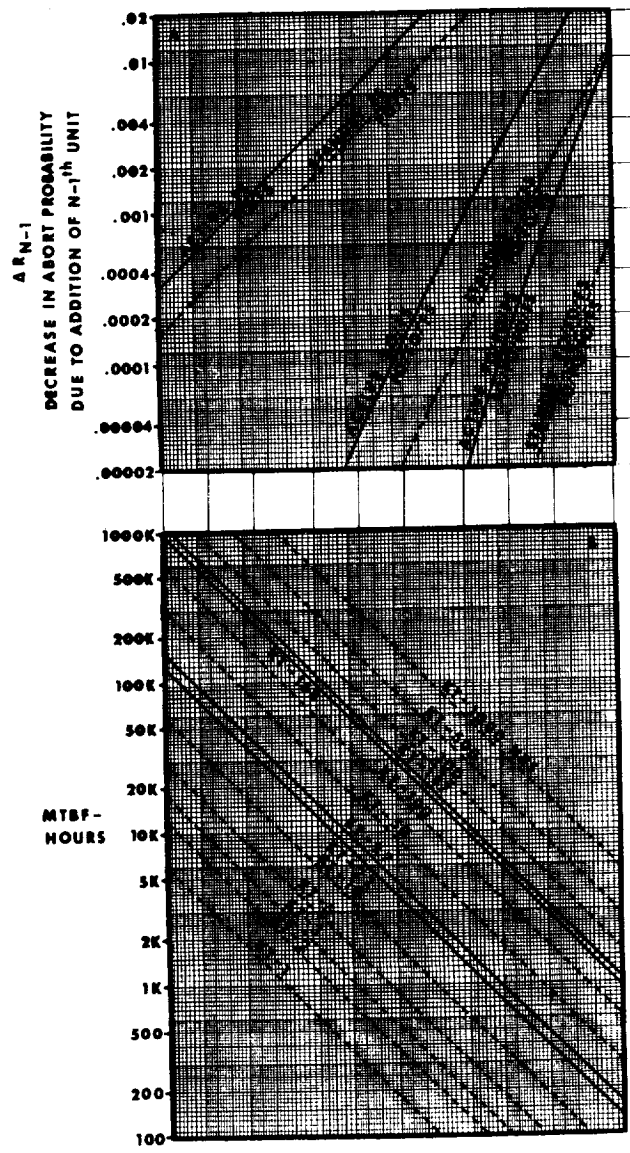
6.8.2 "GO → GO" Redundancy Criterion Nomograph

Contained in Figure 14 is the nomograph for testing whether additional redundancy is economical if both the present level being considered and the previous level resulted in "GO" decisions subsequent to a malfunction on the launch pad. An illustration of the use of this nomograph is described below, using a skeletal outline of the nomograph contained in Figure 15.

The application of this nomograph requires an iterative process, since the variables on each side of the inequality are not independent. After establishing that the addition of a redundant unit which brounght launch malfunctions from "NO GO" to "GO" decisions was economical to add, this criterion applies. Assume that the next additional redundant unit is the optimal level and use this level where all calculations are required. If, in applying the procedure below, the answer is not as assumed then further iterations are required.

- Step 1 - Enter Chart L with the weight of the redundant unit and the vehicle growth cost for the fleet per lb of weight added. Draw a vertical line from the abscissa to the vehicle growth cost per pound of redundancy line and at the intersection draw a horizontal line into Chart I.

- Step 2 - Find the intersection of the horizontal extension from Chart L and the equipment cost per fleet for one more redundant unit for each vehicle. From the intersection draw a vertical line into Chart H.

.00002
.00004
.00006
.0002
.0004
.0006
.002
.004
.006
.02

ΔR_{N-1}
DECREASE IN ABORT PROBABILITY

$1-P_N(F)$

**PROBABILITY OF NO PRE-LAUNCH FAILURE
WITH N UNITS OF EQUIPMENT
FROM COMPUTATION NOMOGRAPH-CHART O)**

.90 .85 .80 .75 .70 .65 .60
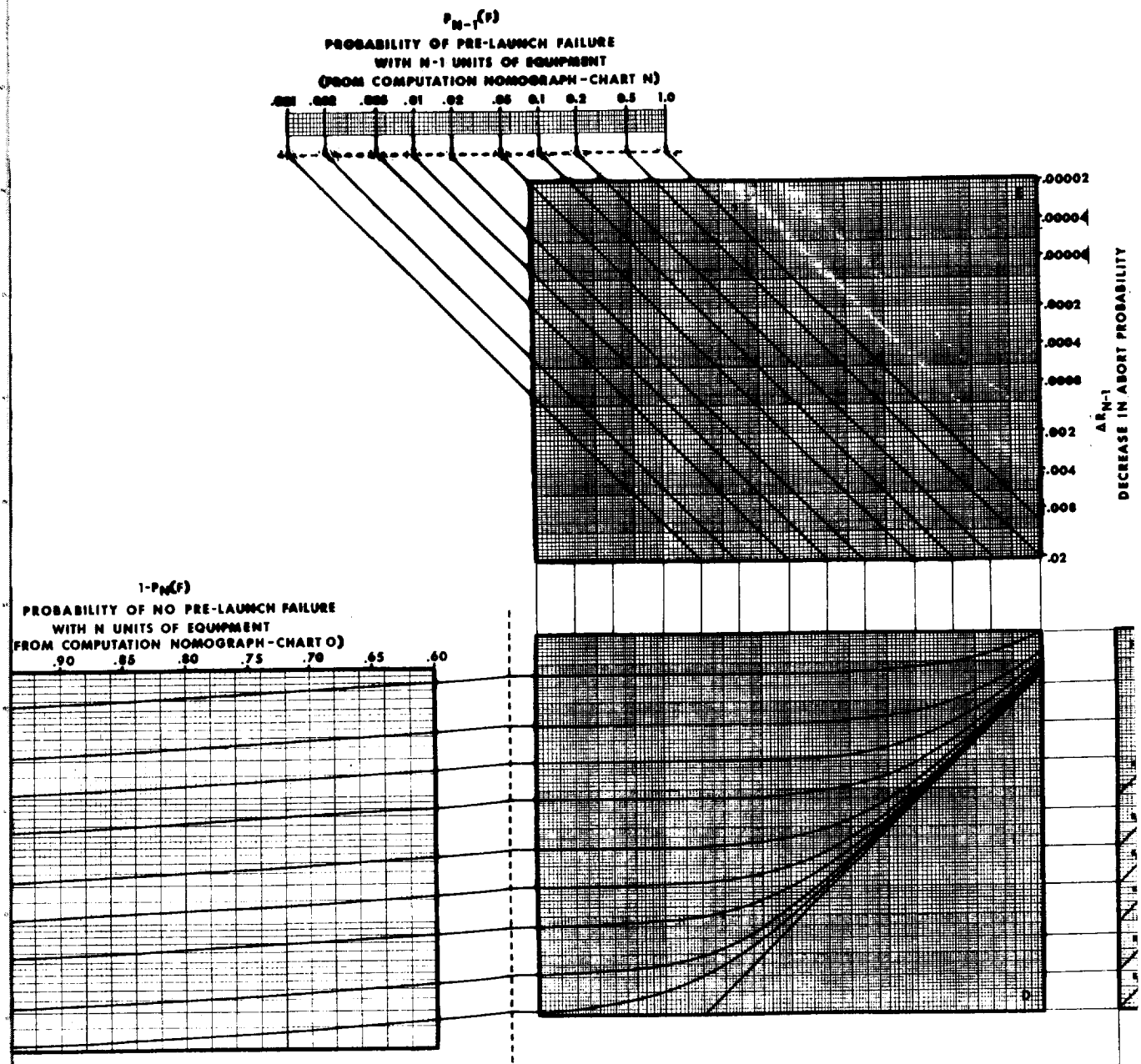
Figure 14

FOLDOUT FRAME 2

INTEGRATED REDUNDANCY SELECTION CRITERIA

"GO→GO" NOMOGRAPH

GRUMMAN

DUE TO ADDITION OF N-1ᵗʰ UNITS
(FROM CHART A)

GROUND COST PER MISSION -
MILLIONS OF DOLLARS

100
50
20
10
5
2
1.0
.5
.2
.1

20 K
50 K
100 K
200 K
500 K
1000 K
2000 K
5000 K

0.1 0.2 0.5 1.0 2 5 10 20 50 100 200 500 1000
WEIGHT OF REDUNDANCY - POUNDS
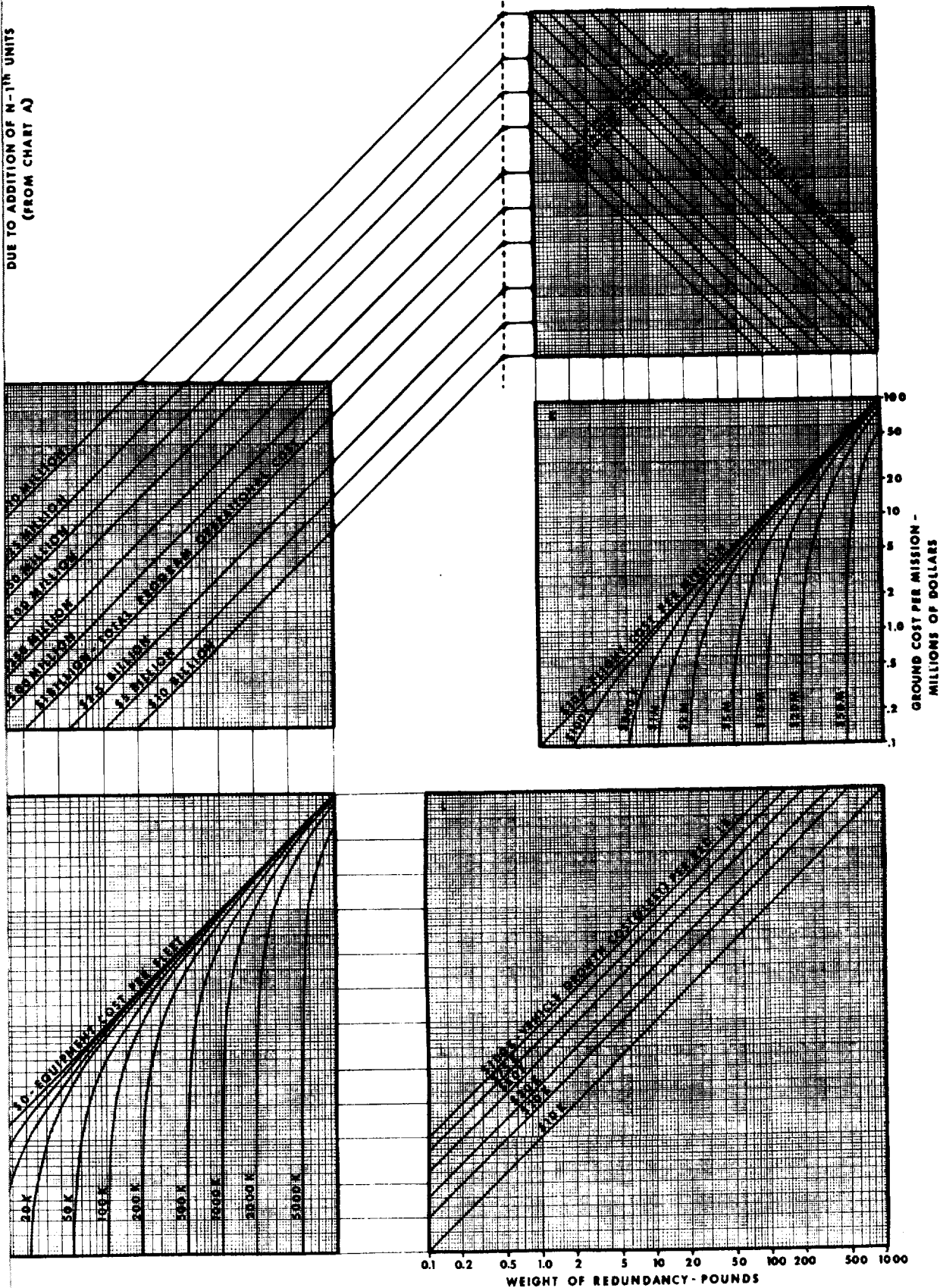
FIGURE 15 — ILLUSTRATION OF USE OF "GO $\longleftrightarrow$ GO" NOMOGRAPH

- Step 3 - Enter Chart K with the ground cost per mission on the ordinate and find the intersection with the flight cost per mission. Draw a vertical line from this intersection into Chart J.

- Step 4 - In Chart J find the intersection of the number of flights in program and the vertical line extended from Chart K. Draw a horizontal line from here to the dashed line on the left of chart and then extend this using the transitional grid into Chart H.
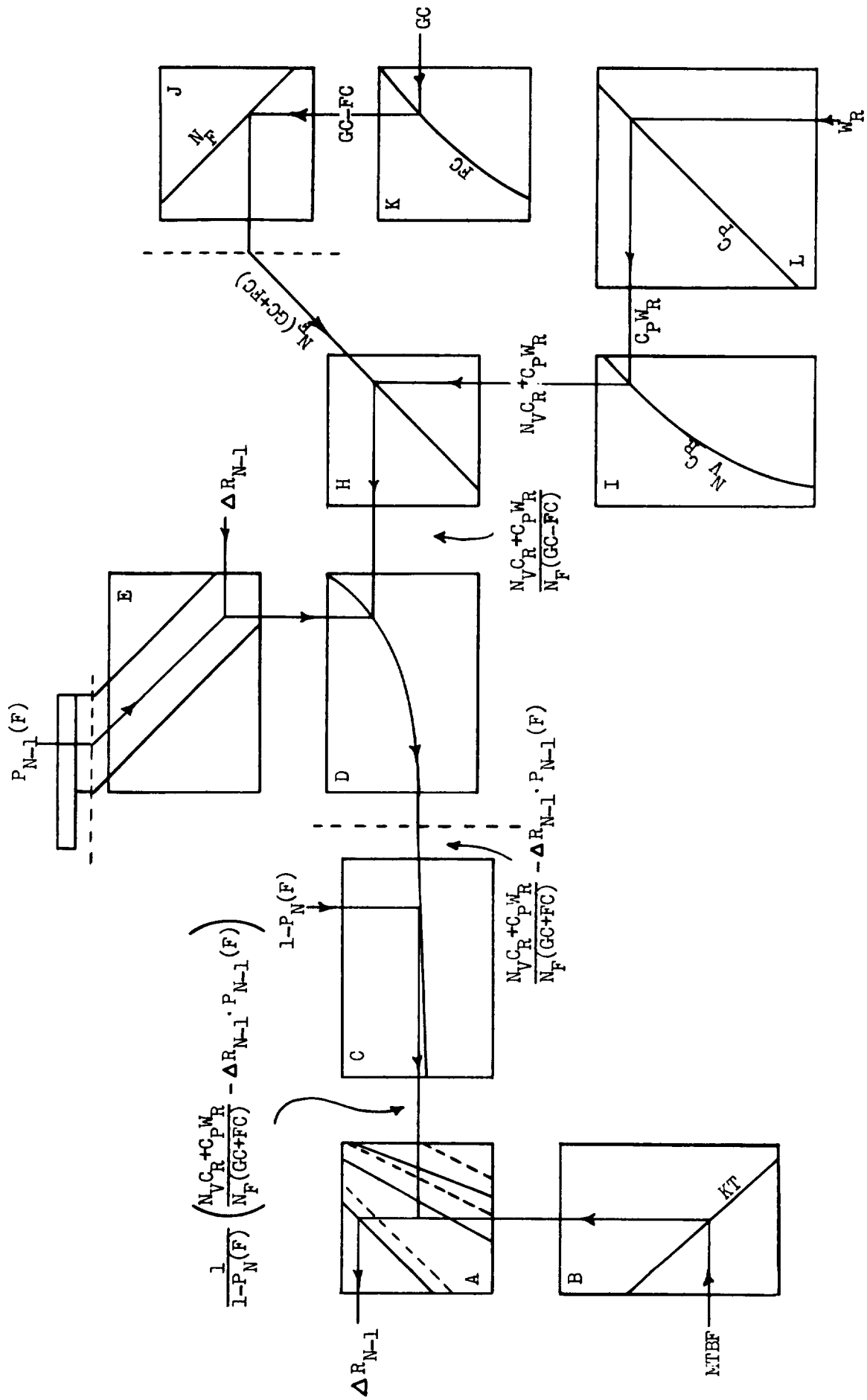
- Step 5 - In Chart H find the intersection of the vertical line from Chart I and the diagonal line from Chart H and draw a horizontal line into Chart D.

- Step 6 - Using Charts B and N on the computation chart, Figure 12, calculate the probability of a pre-launch failure, $P_{N-1}(F)$. Also, using Charts A and B on this nomograph calculate ΔR_{N-1}. This factor, ΔR_{N-1}, is calculated by entering Chart B with the MTBF on the ordinate and intersecting with the flight KT diagonal value. A vertical line is drawn from the intersection and extended into Chart A until it intersects the line representing one less unit than is presently being considered. The value on the ordinate is the value of ΔR_{N-1} to be inputed to Chart E.
  The intersection of ΔR_{N-1} on the ordinate and the $P_{N-1}(F)$ on the diagonal is projected vertically into Chart D.

- Step 7 - The intersection of the horizontal line from Chart H and the vertical line from Chart E specifies the difference between these two terms. The curve in Chart D specified by the intersection is followed out the left hand side and extended into Chart C via the transitional grid.
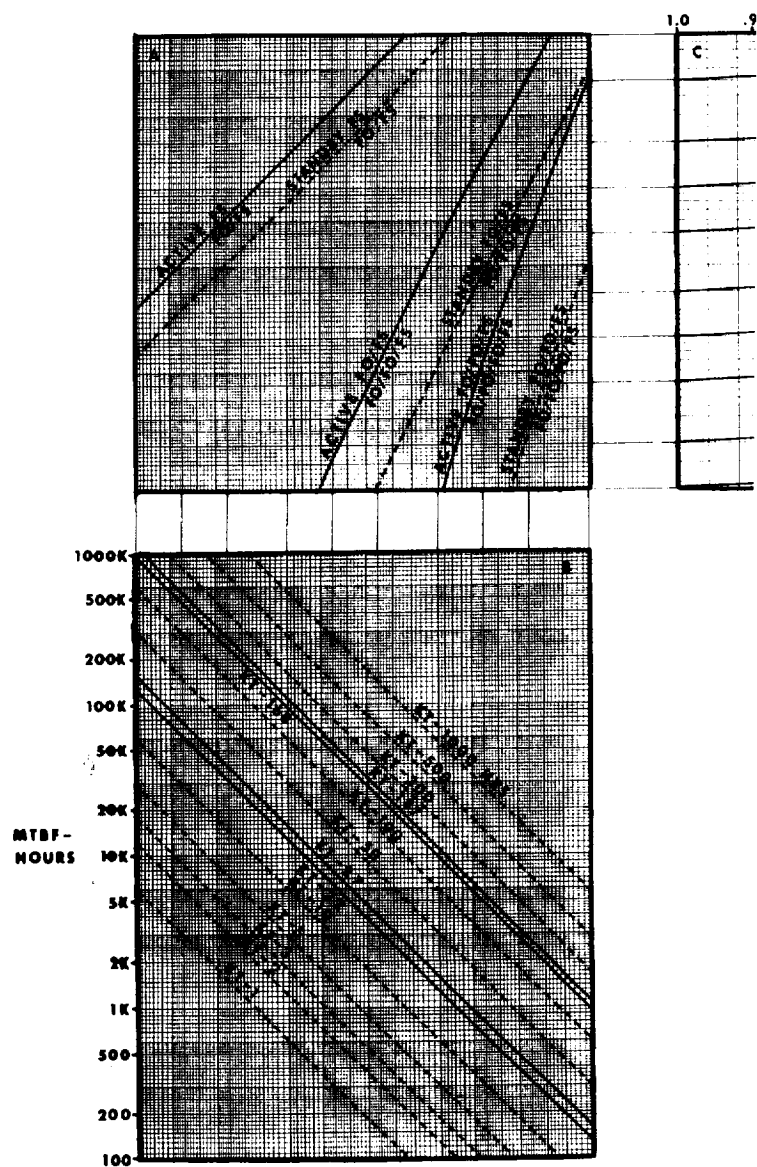
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- Step 8 - Enter Chart C on the abscissa with the probability of no malfunction occurring on the launch pad $(1-P_N(F))$. This quantity is obtained from the computation nomograph on Charts B and O. The intersection of the abscissa value with the line extended from Chart D is projected horizontally into Chart A.

- Step 9 - In Chart B find the intersection of the single unit MTBF and the flight KT value represented by one of the parallel diagonal lines. Project this vertically into Chart A.

- Step 10 - In Chart A find the intersection of the lines extended from Charts B and C. The zone in which these intersect specifies the redundancy level if it coincides with the assumed optimal level. If the decision zone specified a higher level than originally assumed then repeat the process, increasing the redundancy. If the decision zone specified a lower level than originally assumed then repeat the process using a lower level of redundancy or terminate the process if it is lower than the level specified by the "NO GO → GO" redundancy criterion.

6.8.3 NO GO → GO Redundancy Criterion Nomograph

The nomograph for applying this criterion is presented in Figure 16. It is applied when the addition of a redundant unit eliminates delays on the launch pad subsequent to a malfunction. A skeletal outline of the nomograph is contained in Figure 17 and a description for its use is below.

- Step 1 - Establish, by using the "GO/NO GO" launch criterion nomograph contained in Figure 13, the two adjacent levels of redundancy which have the attributes of "NO GO" and "GO."

- Step 2 - Repeat Steps 1 thru 5 of Section 6.8.2 which covers Charts H, I, J, K, and L. The resultant should be a horizontal line projected into Chart D.
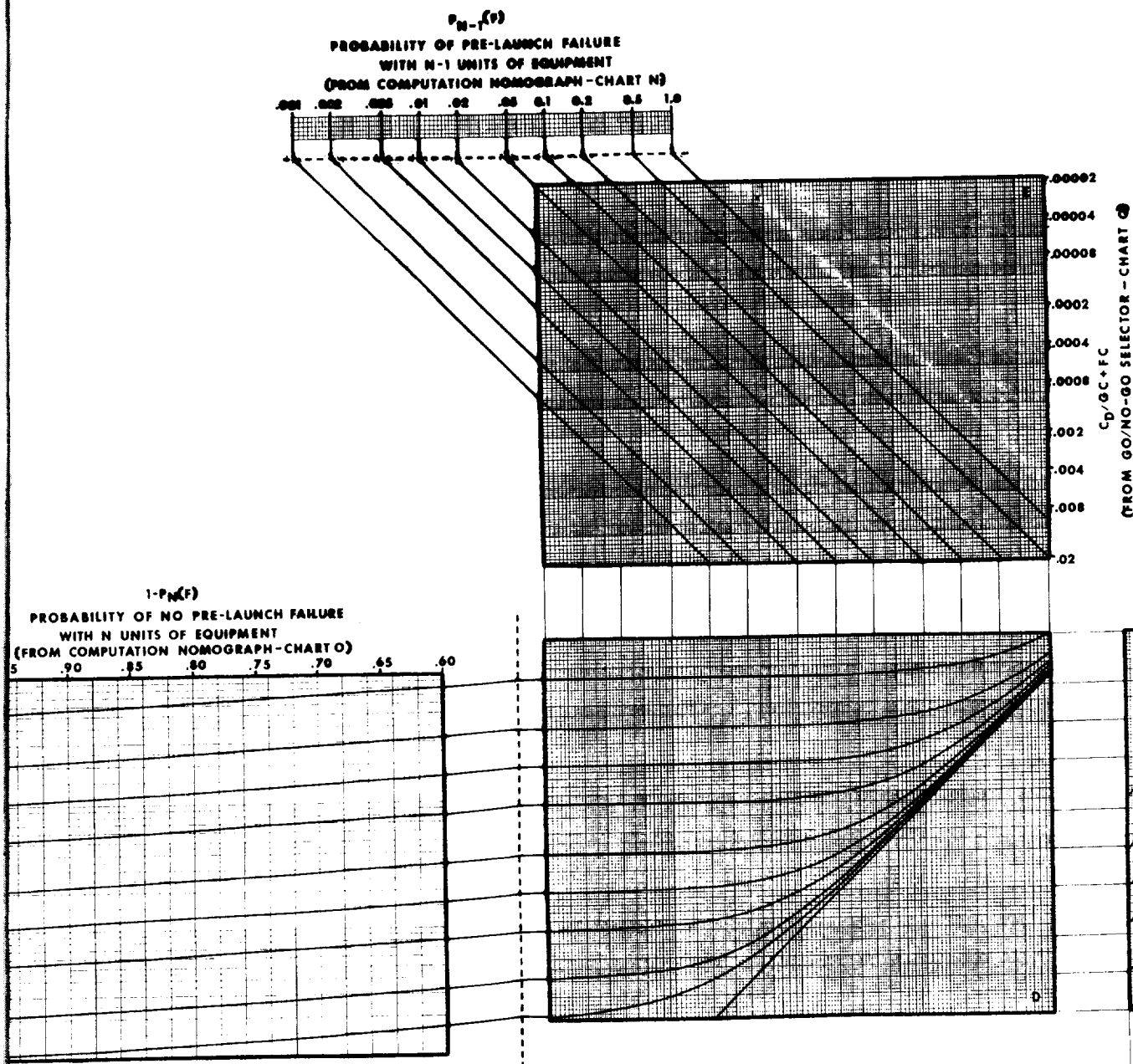
Figure 16

INTEGRATED REDUNDANCY SELECTION CRITERIA

"NO GO → GO" NOMOGRAPH

Nomograph charts with the following labels:

Upper-right chart (M): GROUND EQUIPMENT DEVELOPMENT COST

Left-center chart (N):
- $10 MILLION
- $25 MILLION
- $50 MILLION
- $100 MILLION
- $250 MILLION
- $500 MILLION
- $1 BILLION — TOTAL PROGRAM OPERATIONAL COST
- $2.5 BILLION
- $5 BILLION
- $10 BILLION

Right-center chart (K):
- $100K — FLIGHT COST PER MISSION
- $500K
- $1 M
- $2 M
- $5 M
- $10 M
- $25 M
- $50 M

Lower-left chart:
- $10 K — EQUIPMENT COST PER UNIT
- $50 K
- $100 K
- $250 K
- $500 K
- $1000 K
- $2000 K
- $5000 K

Lower-right chart:
- $100 K — VEHICLE DEVELOPMENT COST TO OVER REDUNDANCY
- $500 K
- $1 M
- $5 M
- $10 M

Right axis: GROUND COST PER MISSION — MILLIONS OF DOLLARS
Scale: 100, 50, 20, 10, 5, 2, 1.0, .5, .2, .1

Bottom axis: WEIGHT OF REDUNDANCY - POUNDS
Scale: 0.1, 0.2, 0.5, 1.0, 2, 5, 10, 20, 50, 100, 200, 500, 1000

FIGURE 17 — ILLUSTRATION OF USE OF "NO-GO → GO" NOMOGRAPH

Figure 18

INTEGRATED REDUNDANCY SELECTION CRITERIA
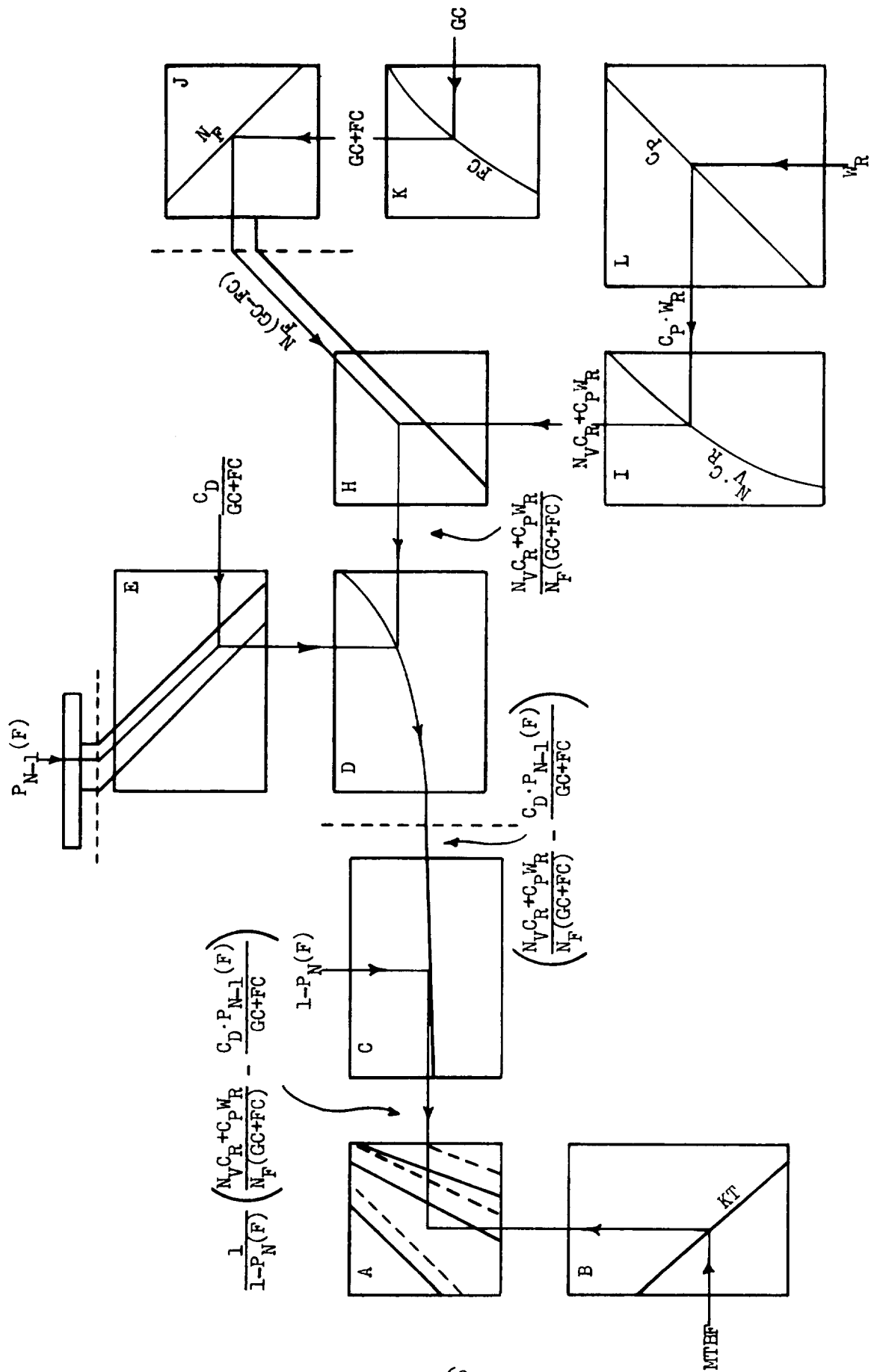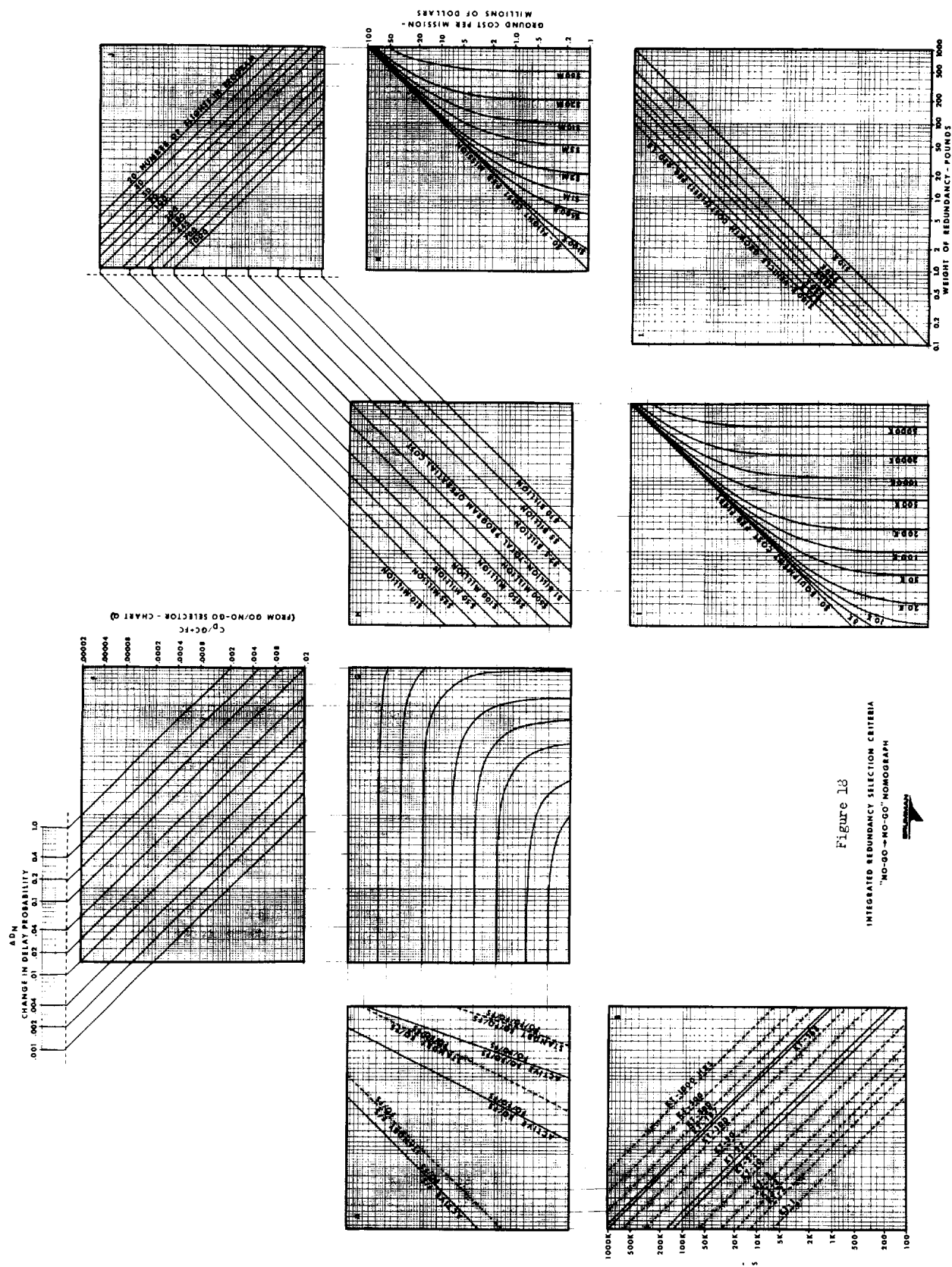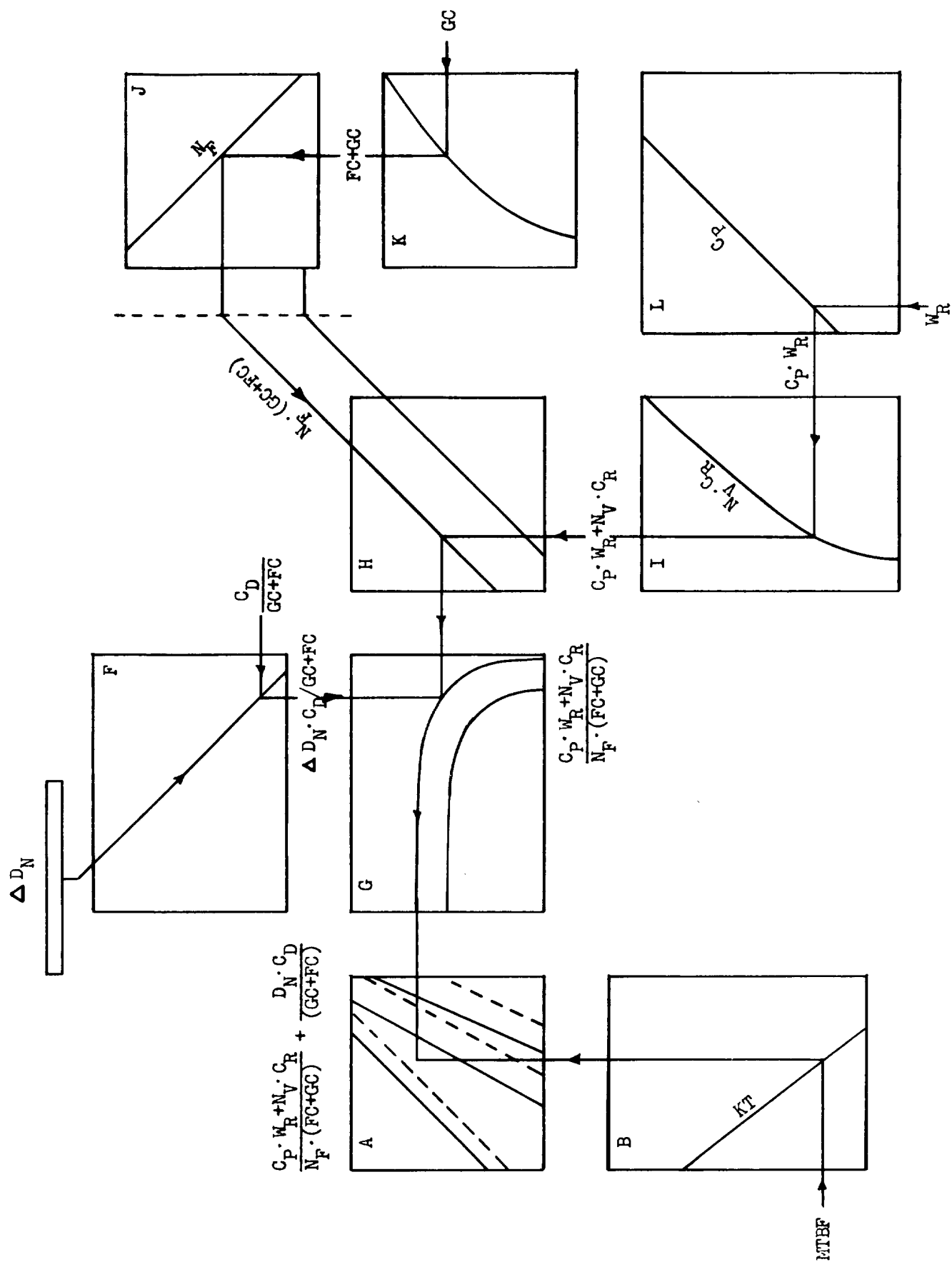"NO-GO→NO-GO" NOMOGRAPH

FIGURE 19 — ILLUSTRATION OF USE OF "NO GO → NO GO" NOMOGRAPH

and procurement cost divided by the total nominal mission operational cost.

- Step 2 - On the computation nomograph, using Charts B, M, and N, establish the change in delay probability (ΔD_N) due to addition of the assumed optimal level of redundancy. Also, on the "GO/NO GO" launch criterion nomograph using Charts K, Q, and R calculate the cost of a delay divided by the single mission cost ($C_D/GC + FC$).

- Step 3 - Enter Chart F on the diagonal corresponding to the ΔD_N value calculated in Step 3 and on the ordinate with the value computed for ($C_D/GC + FC$). At the intersection of the ordinate value and ΔD_N value project a line vertically into Chart G.

- Step 4 - In Chart G determine the intersection of the horizontal and vertical lines from Charts H and F respectively. This intersection is the sum of these two values. Follow the appropriate curve in Chart G out the left hand side and project it horizontally into Chart A.

- Step 5 - Enter Chart B with the MTBF value for a single unit of this equipment and intersect the diagonal which represents the flight KT factor. At the intersection, draw a vertical line into Chart A.

- Step 6 - The intersection of the horizontal and vertical lines extended from Charts G and B respectively, in Chart A, specifies the redundancy zone. This redundancy level, if coincidental with the assumed optimal solution, is the most economical level and no further analysis need be conducted.

If the redundancy level is other than the assumed then perform the analysis again assuming a different level of redundancy is optimal. Use a greater quantity if the intersection zone is greater than the assumed and use a lesser amount if the intersection zone is less than that assumed.
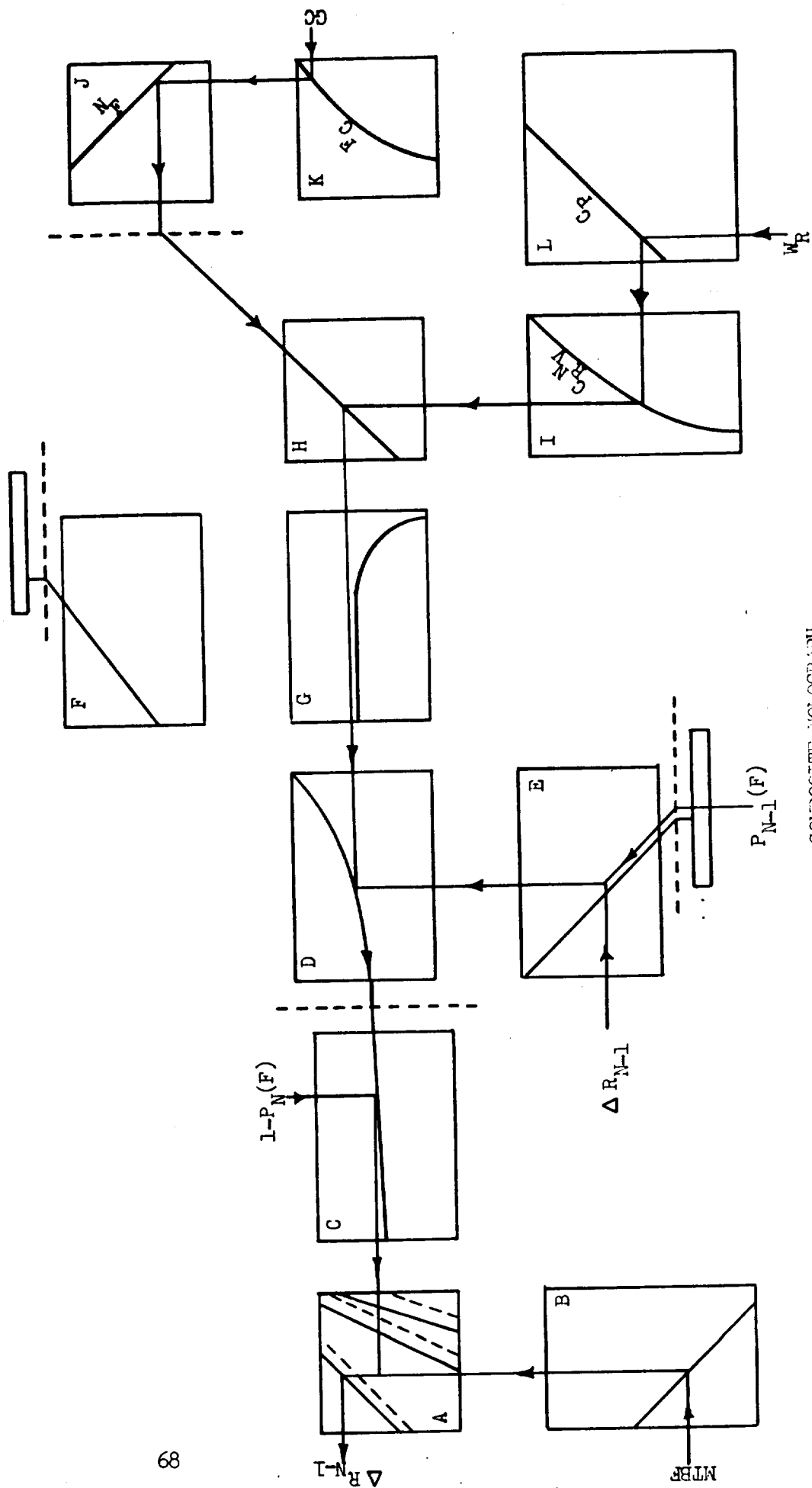
6.8.5 Composite Nomograph

The composite nomograph Figure 11, contains all the charts that were used in the individual criterion nomographs. Thus, any single criterion can be evaluated and the procedure to be followed is similar to those specified in Sections 6.8.2 thru 6.8.4. A couple of transistions through unused charts need clarification. The skeletal outline of the nomograph shown in Figure 20 thru 22 together with the explanation below highlights these differences.

- "GO → GO" Redundancy Criterion - In applying this criterion on the composite nomograph follow the procedure outlined in Section 6.8.2 using Charts A, B, C, D, E, H, I, J, K, and L. Ignore Charts F and G. When exiting from Chart H extend the line horizontally through Chart G, without performing any operation, directly into Chart D.

- "NO GO → GO" Redundancy Criterion - In applying this criterion on the composite nomograph follow the procedure outlined in Section 6.8.3 using Charts A, B, C, D, E, H, I, J, K, and L. Follow the deviations outlined in above paragraph on "GO → GO" Redundancy Criterion.

- "NO GO → NO GO" Redundancy Criterion - In applying this criterion on the composite nomograph follow the procedure outlined in Section 6.8.4 using Charts A, B, F, G, H, I, J, K, and L. Ignore Charts C, D, and E. The output from Chart G should pass horizontally through Chart D until reaching the dashed line outside left hand border. At this point follow the transistion grid through Chart C to the left border. Then make a horizontal projection into Chart A. No other deviations are required.

6.8.6 Computation Nomograph

The computation nomograph, Figure 12, is used in conjunction with all three criterion. The quantities $P_{N-1}(F)$ and and $1-P_N(F)$ are used in both the "GO → GO" and "NO GO → GO" redundancy criterion, whereas the quantity ΔD_N is

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COMPOSITE NOMOGRAPH

FIGURE 20 -- ILLUSTRATION OF "GO $\rightarrow$ GO" CRITERION
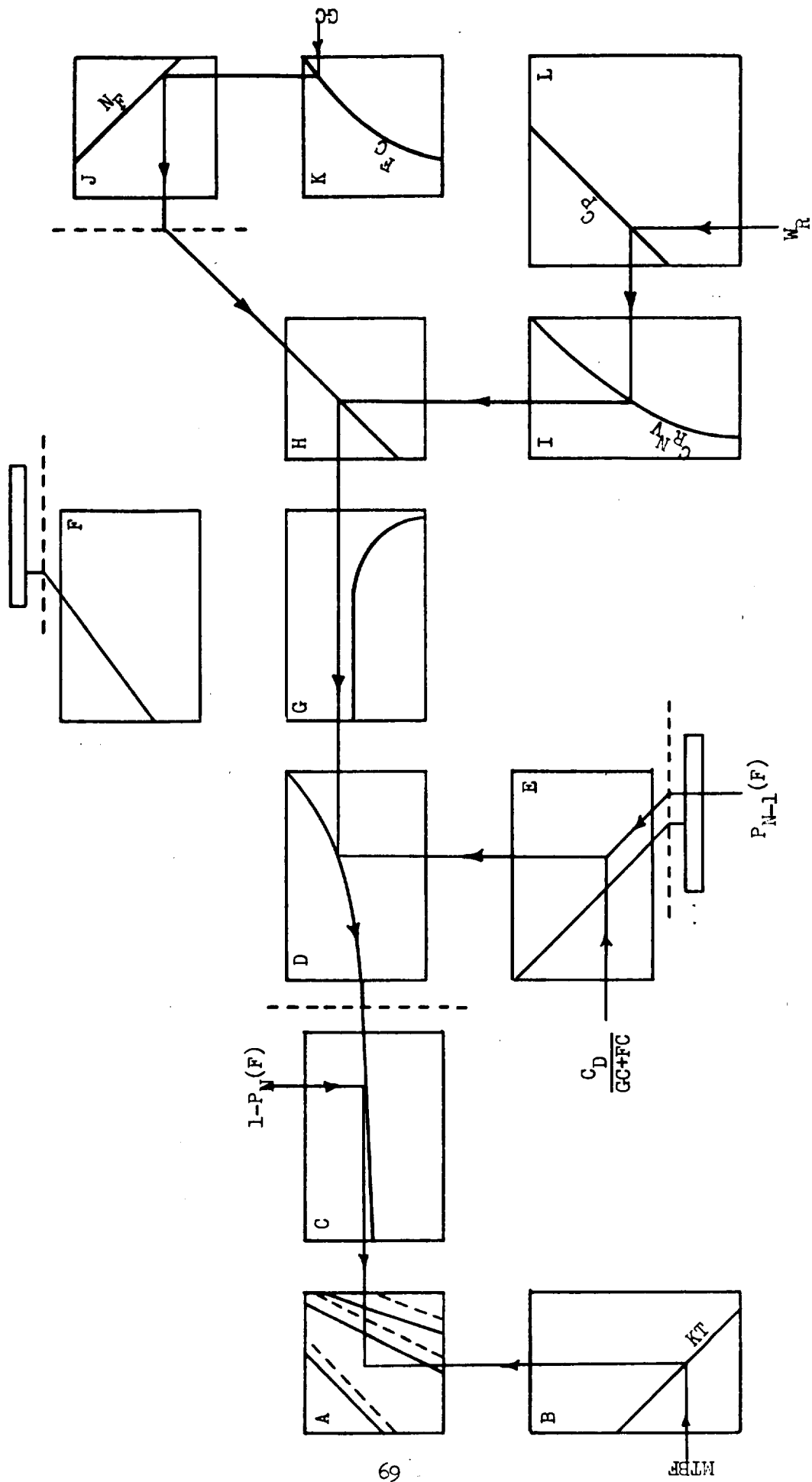
COMPOSITE NOMOGRAPH

FIGURE 21 - ILLUSTRATION OF "NO GO → GO" CRITERION

COMPOSITE NOMOGRAPH

FIGURE 22 - ILLUSTRATION OF "NO GO → NO GO" CRITERION

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used in the "NO GO → NO GO" redundancy criterion. Figure 23 thru 25 skeletal outline of the nomograph and used in conjunction with the paragraphs below will depict the procedure.

- Procedure for Calculating $P_{N-1}(F)$ - (Refere to Figure 23)

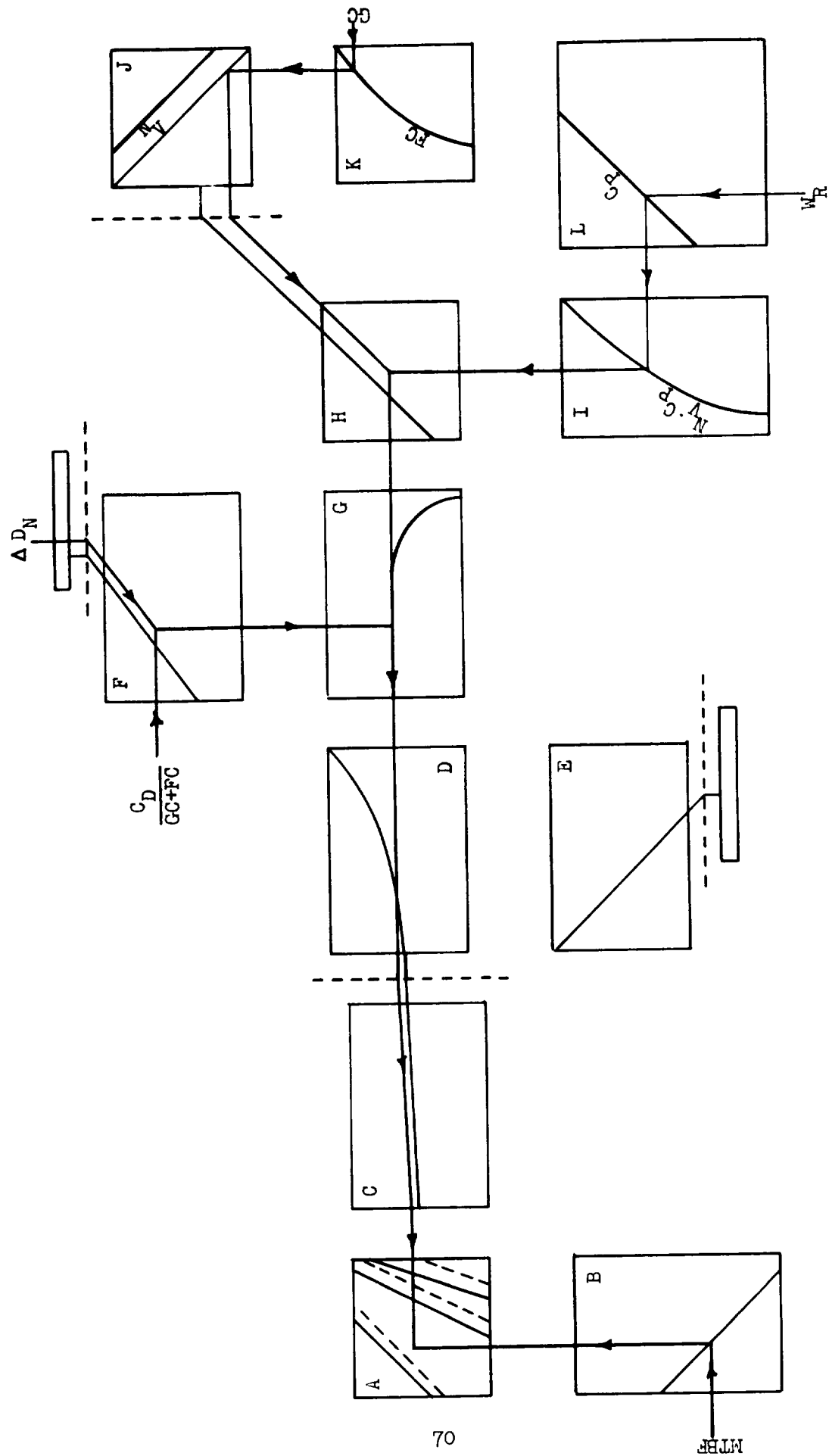 This quantity represents the probability of a failure during pre-launch operations if there were one less unit of equipment (N-1) then presently under consideration.

  - Step 1 - Enter Chart B with the single unit MTBF on the ordinate and the pre-launch operations KT factor represented by one of the diagonal lines. At the intersection of the ordinate value and diagonal line draw a vertical line into Chart N.

  - Step 2 - In Chart N find the intersection of the vertical line extended from Chart B and the diagonal line representing the level of redundancy presently under consideration. The ordinate value, denoted on the right edge of the chart, corresponding to this inter-section is the probability of failure.

- Procedure of Calculating $1 - P_N(F)$ - (Refer to Figure 24)

  This quantity represents the probability of no malfunction occurring during pre-launch operations with N units of equipment (the quantity presently being economically evaluated).

  - Step 1 - Same as Step 1 for $P_{N-1}(F)$ except extend vertical line into Chart O.

  - Step 2 - Locate the intersection of the line extended from Chart B with the curved line in Chart O representing the present level of redundancy under consideration. The value of $1 - P_N(F)$ on the absissa corresponding to this intersection is the probability of no malfunction during pre-launch operations.
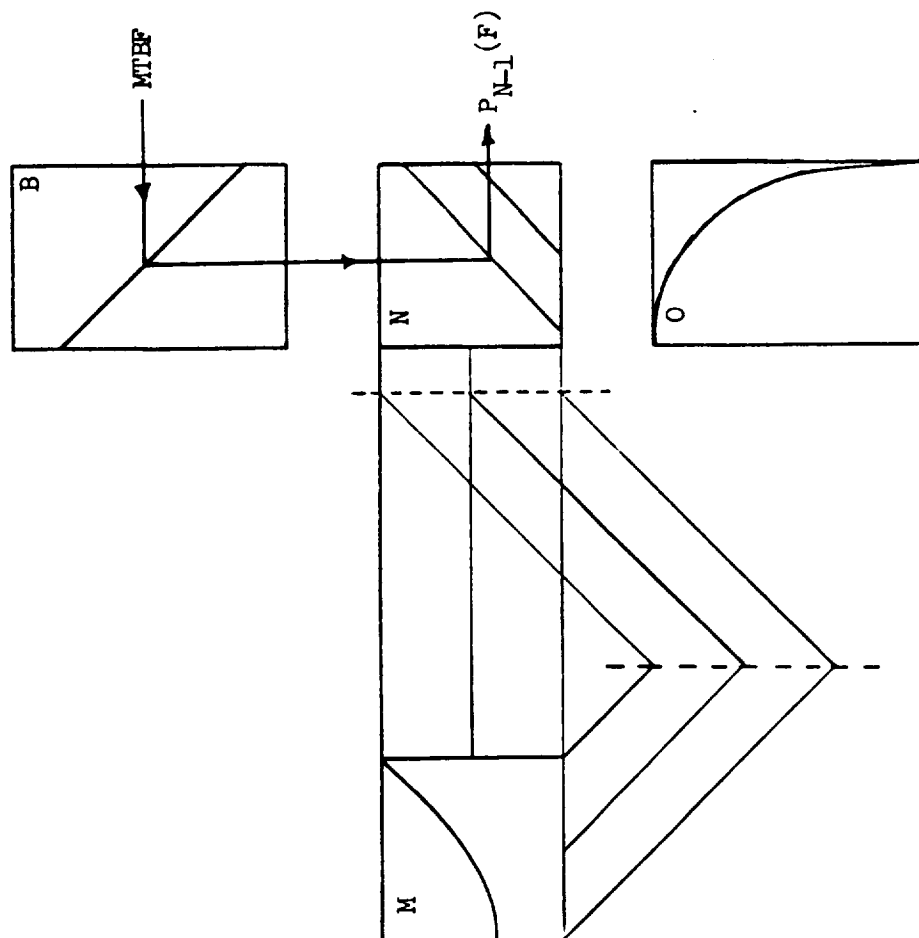
COMPUTATION NOMOGRAPH

FIGURE 23 - ILLUSTRATION OF PROBABILITY OF PRE-LAUNCH FAILURE ($P_{N-1}(F)$) CALCULATION
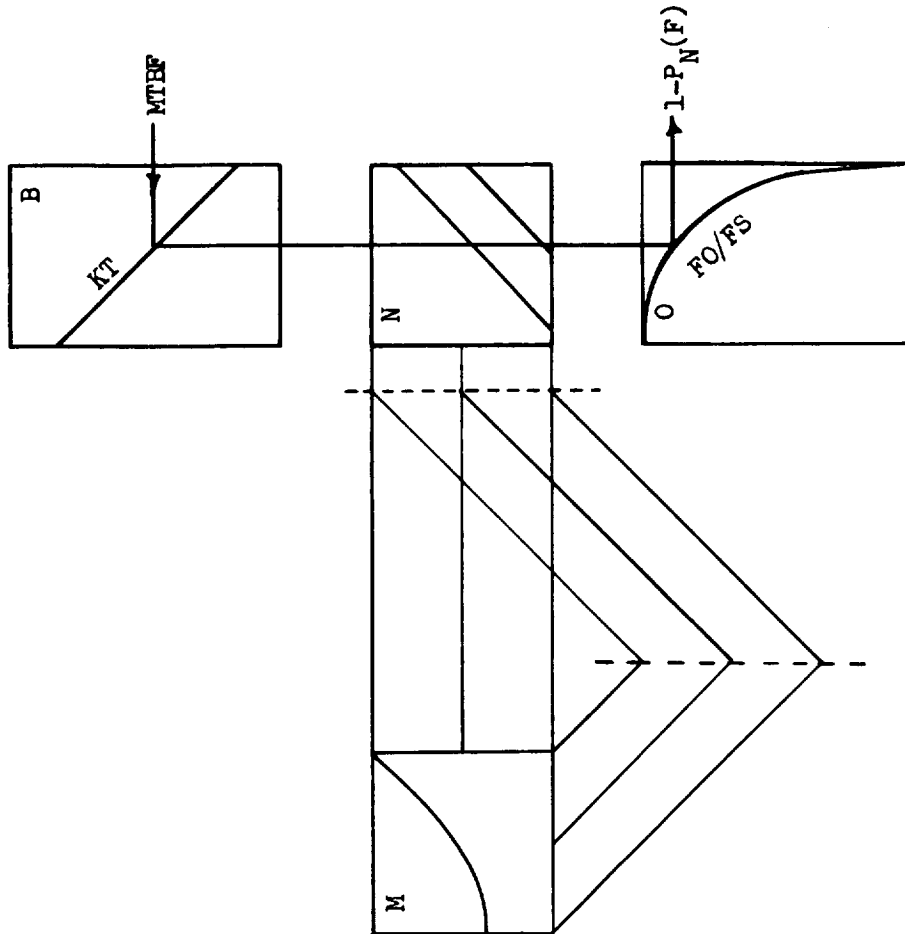
COMPUTATION NOMOGRAPH

FIGURE 24 - ILLUSTRATION OF PROBABILITY OF NO
PRE-LAUNCH FAILURE $(1-P_N(F))$ CALCULATION

- Procedure for Calculating ΔD_N - (Refer to Figure 25)

  This quantity represents the change in delay probability due to addition of the Nth unit of equipment. If the form of redundancy is standby then the change in delay probability is zero. Thus, this computation is only needed if an active form of redundancy is being considered.

  - Step 1 - Repeat Step 1 of procedure for calculating $P_{N-1}(F)$.

  - Step 2 - In Chart N locate the intersection of the vertical line extended from Chart B with the diagonal labelled with the present level of redundancy being considered. At this intersection project a horizontal line out the left side to the dashed line. Then follow the transitional grid labelled smaller value to Chart M. At the edge of Chart M on the abscissa extend a vertical line into the chart.

  - Step 3 - In Chart N locate the intersection of the vertical line extended from Chart B with the diagonal labelled with one more unit than presently under consideration. Project this intersected point horizontally into Chart M following the larger value grid.

  - Step 4 - In Chart M locate the intersection of the vertical and horizontal lines extended from Chart N. The value of D_N is specified by the curved line in Chart M at the intersection.
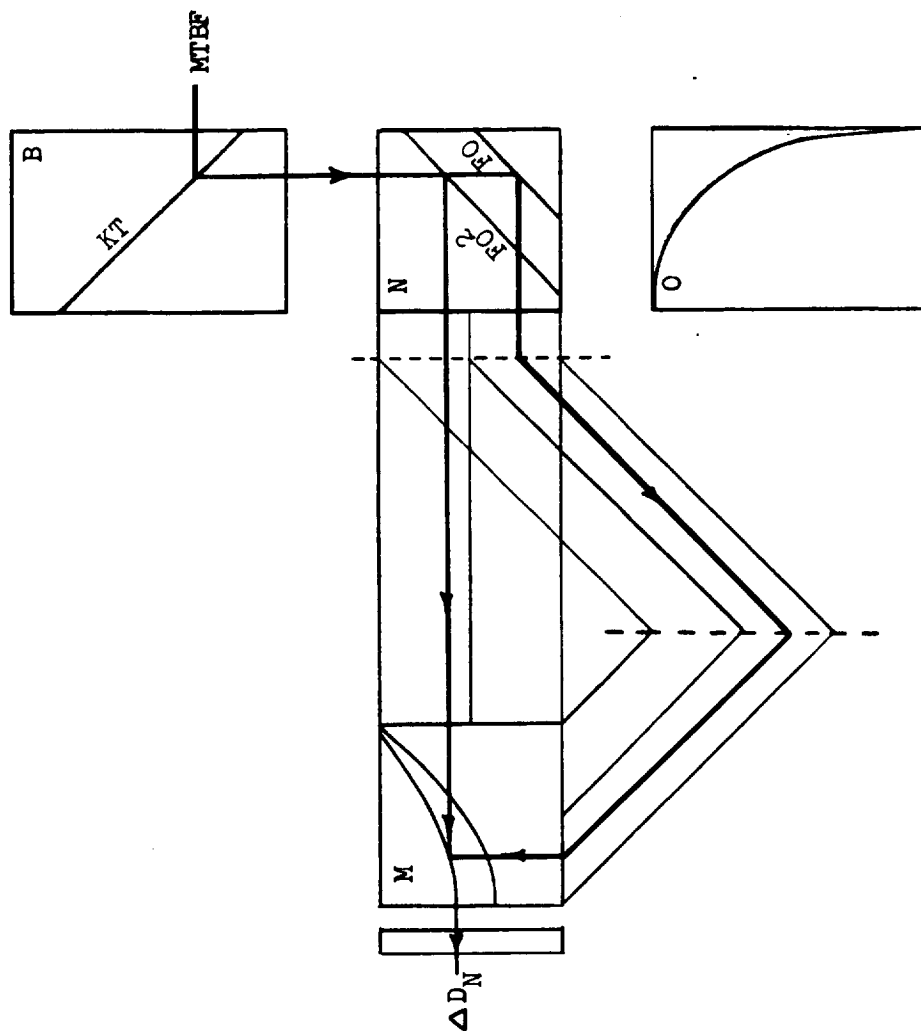
COMPUTATION NOMOGRAPH

FIGURE 25 - ILLUSTRATION OF DELAY PROBABILITY (ΔD_N) CALCULATION

6.8.7 "GO/NO GO" Launch Nomograph

This nomograph, Figure 13, is used for two purposes; evaluating the economics of launching or delaying the vehicle subsequent to a malfunction and computing the factor $C_D/GC + FC$ for use in the "NO GO → NO GO" and the "NO GO → GO" criterion evaluations. A skeletal outline of this nomograph with an illustration of its use is contained in Figure 26. Section 5 and Appendix II of this report contains a comprehensive discussion of this criterion. The initially developed nomograph contained therein has been modified slightly for use with the integrated redundancy criterion

Procedure for Computing $C_D/GC + FC$

This computation involves Charts K, Q and R and is an input to Chart E for the NO GO → GO criterion and to chart F for the NO GO → NO GO criterion.

- **Step 1** - The nominal ground operations cost for a single flight located on the abscissa of Chart K is summed with the flight operations cost, one of the family of curved lines, by locating the intersection of these values. A horizontal line is extended to the right, from the intersection points and, via the transitional grid, extended into Chart Q.

- **Step 2** - The anticipated hours of delay required to repair the malfunction is entered on the ordinate of Chart R and intersected with the appropriate cost per hour diagonal line. At the intersection a vertical line is drawn into Chart Q.

- **Step 3** - In Chart Q the extended diagonal line from Chart K is intersected with the vertical projection from Chart R. A horizontal line is extended from this intersection to the right border of Chart Q. The value specified on the ordinate is the input required for Charts
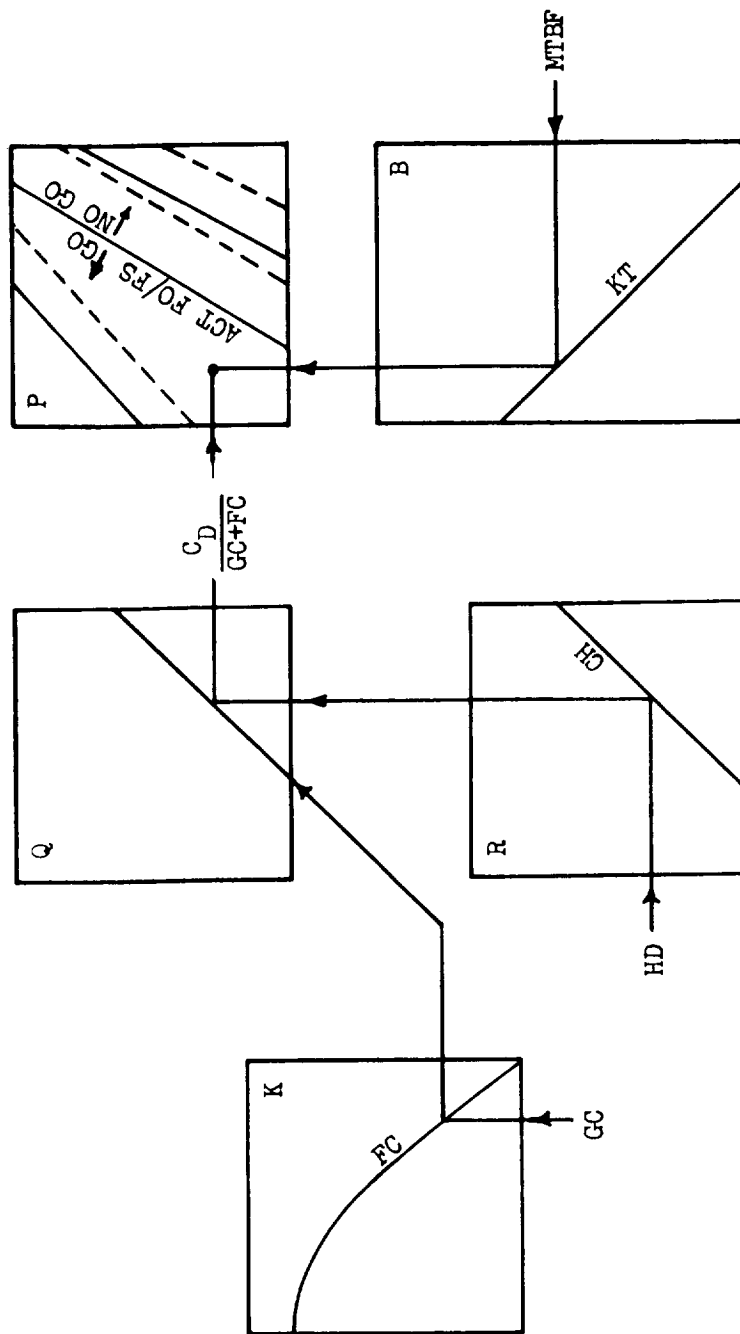
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FIGURE 26 - ILLUSTRATION OF "GO/NO-GO" LAUNCH CRITERION
AND CALCULATION OF $C_D/GC+FC$

E and F of the appropriate individual criterion nomograph. It should

be noted that the scale is logarithmic for purposes of interpolation.

Procedure For Identifying "NO GO → GO" Redundancy Level

The procedure for selecting the optimal level of redundancy commences

with the first level of redundancy which eliminates delays. Chart P enables

this level to be identified as follows:

- Step 1 - Repeat steps 1, 2, and 3 for computing C_D/GC + FC. Extend

  the resultant of step 3 horizontally into Chart P.

- Step 2 - In Chart B find the intersection of the mean time between

  failures (MTBF) for a single unit on the ordinate and the flight KT

  factor which is one of the diagonal lines. Draw a vertical line

  from this intersection into Chart P.

- Step 3 - In Chart P find the intersection of the vertical and horizontal

  lines from Charts B and Q respectively. The zone in which this inter-

  section occurs specifies the two adjacent redundancy levels for which

  the launch decision changes from "NO GO" to "GO," subsequent to a

  launch pad malfunction.

  The line above and to the left is the NO GO level and the line below

  and to the right is to GO level.

6.9 APPLICATION OF INTEGRATED REDUNDANCY SELECTION CRITERIA

The integrated redundancy selection criteria was applied to the several

shuttle equipments, shown in Table 3 , to be NO GO subsequent to launch pad

malfunction. The illustration included here, of the Inertial Measurement Unit,

clearly shows that it is most economical to have an FO^3/FS configuration when

delay costs are considered. When applying the initial criterion of Section 4

which does not consider the delay costs the recommended level of redundancy

$$\Delta R < \frac{\text{Cost of Delay}}{\text{Cost of a Mission}}$$

$$\Delta R < \frac{32,000}{5 \times 10^6}$$

$$\Delta R < .0064$$

Using Chart P on the "GO/NO GO" launch criterion nomograph identifies the first "GO" level as FO^3/FS, since the value of ΔR for FO^2/FS prior to a failure on the launch pad is .008, and for FO^3/FS, .00043.

Applying the "NO GO → GO" redundancy criterion nomograph in Figure 16. results in it being economical to add the FO^3/FS level of redundancy. Also by applying the "GO → GO" redundancy criterion it is found that the next level, FO^4/FS, is not economical to add.

Using the equation developed for each criterion below, the actual difference in values and program costs can be calculated.

"NO GO → GO" Redundancy Criterion

The inequality which must be satisfied is

$$\Delta R_{FO^3/FS} > \frac{1}{1 - P_{FO^3}(F)} \left(\frac{C_R \cdot N_V + W_R \cdot C_P}{(GC + FC) \cdot N_F} - \frac{C_D \cdot P_{FO^2}(F)}{(GC + FC)} \right)$$

$$\Delta R_{FO^3/FS} > \frac{1}{1 - .075} \left(\frac{(118,000) \cdot (3) + (57) \cdot (16,500)}{(5 \times 10^6) \cdot (445)} \quad \frac{(32,000)(.06)}{5 \times 10^6} \right)$$

$$\Delta R_{FO^3/FS} > .0002138$$

Since $\Delta R_{FO^3/FS}$ is .00043 it is economical to add this redundant unit.

Initial Redundancy Selection Criterion

The inequality which must be satisfied is

$$\Delta R_{FO^3/FS} > \frac{C_R \cdot N_V + W_R \cdot C_P}{(GC + FC) \cdot N_F}$$

$$\Delta R_{FO^3/FS} > \frac{(118,000) \cdot (3) + (57) \cdot (16,500)}{(5 \times 10^6) \cdot (445)}$$

$$\Delta R_{FO^3/FS} > .0005818$$

Since ΔR_{FO^3} equals .00043 this level of redundancy would not be recommended.

The expected program costs are broken down and tabulated below, showing the elemental cost fluctuations.

| Program Cost Element | Redundancy Level | | $\Delta\$$ |
	FO^2/FS	FO^3/FS	$FO^3 - FO^2$
Baseline	Includes Procurement of C_1	Four Units, Design, Test, Etc. C_1	0
Delay	$P_{FO^2}(F) \cdot C_D \cdot N_F$ $\$854,400$	0	$\$-854,400$
Procurement	Included in C_1 0	$\$354,000$	$\$+354,000$
Vehicle Growth	Included in C_1 0	$\$940,500$	$\$+940,500$
Aborts	$P_{FO^2}(A) \cdot (GC+FC) \cdot N_F$ $\$1,009,705$	* $\$123,064$	$\$-886,641$
Total	$C_1 + 1,864,105$	$C_1 + 1,417,564$	$\$-446,541$

$$ * \left\{ P_{FO^3}(A) \cdot (1 - P_{FO^3}(F)) + P_{FO^2}(A) \cdot P_{FO^3}(F) \right\} \cdot (GC + FC) \cdot N_F \}$$

As the table illustrates, a savings during the operational phase of $\$1,741,041$ is realized for an initial investment of $\$1,244,500$. Thus, a net expected program cost savings of $\$445,541$, is realized by placing an FO^3/FS configuration onboard each space shuttle rather than an FO^2/FS system.

7.0 CONCLUSIONS

In the course of this study it has become evident that the use of an economic criterion for redundancy selection may result in significant operational cost savings as well as Shuttle total program cost savings. Prior redundancy selection criteria that did not consider the operational aspects, such as delay costs, were shown to be inadequate for some equipments but generally give solutions similar to the integrated approach. The prior approaches to redundancy selection were simpler but the few cases where the complicated integrated criterion results in different solutions, the significant dollar savings warrants its application.

The integrated redundancy selection criterion has been synthesized by screening out several cost impacts whose sensitivity and magnitude were shown to be dwarfed by the procurement, vehicle growth, abort and delay costs. Thus, the integrated criterion is a good practical working tool, with inaccuracies which are generally insignificant.

Nomographs have been developed to expedite the analysis, provide table top computation, and quick sensitivity analysis. However, these tools are cumbersome to apply when many iterations of numerous configurations are required. Therefore, it would be beneficial to develop a computer program for general application.

These redundancy selection and operational criterion have been developed solely to assess the economic consequences of various approaches. In addition there are other non-economic and qualitative factors that must be weighed in the total decision process. Thus, these criterion are not meant to be a decision panacea but only an aid to management in rendering judicious decisions.

APPENDIX I

DESCRIPTION OF INITIAL REDUNDANCY SELECTION
CRITERION NOMOGRAPH AND PROCEDURE FOR ITS USE

(A) Description of Nomograph

The nomograph shown in Figure 4 of Section 4.3 is the updated version of

a similar one developed at Grumman for use in our in-house Shuttle analysis.

This figure displays two criteria; cost and payload. Chart A, which does not

pertain to the cost effectiveness criterion, will not be discussed and should

be ignored. This chart is used for making payload effectiveness decisions and

is not included under the scope of this study.

The function of each chart is as follows:

• Chart D multiplies the weight of the redundant unit and its installa-

  tion hardware (ΔW) times the cost to incorporate each pound of added

  redundancy into the fleet (CPI). CPI should include all secondary or

  propagative costs such as equipment design costs, increased power

  costs, increased cooling costs, etc., and should be in units of the

  cost per pound of redundancy for the entire fleet. The resultant of

  this multiplication, (ΔW) . (CPI), is the total cost of carrying the

  installed redundancy.

• Chart E adds the cost of procuring the redundant units, which is the

  purchase cost of one redundant unit (CR) times the number of units

  purchased (normally the number of vehicles in the fleet-NV) to the

  cost of carrying the installed redundancy (ΔW x CPI) obtained from

  Chart D. The resultant, (CR) (NV) + (ΔW) (CPI), is the total cost

  of adding redundancy.

- Chart G adds the ground cost per mission (GC) to the flight cost per mission (FC) to obtain the total operational cost per mission (GC + FC). This total cost per mission is the wasted cost of an aborted flight. Therefore, all operational ground costs of a mission should be included such as: inspection and post flight checkout of vehicle, pre-launch checkout, fueling and launch operations. The flight cost per mission should include all post launch operational costs including communication and tracking costs, Mission Control operational costs and other flight support costs.

- Chart H multiplies the total operational cost per mission, (GC) + (FC), obtained from Chart G times the number of flights in the program (NF). The resultant, (GC + FC) (NF), is the total operational cost of the program.

- Chart F . divides the total operational cost of the program, (GC + FC) NF, obtained from Chart H by the total cost of adding redundancy, (CR) (NV) + (ΔW) (CPI), which is obtained from Chart E. The resultant is the right side of the decision inequality:

$$\Delta R > \frac{(CR)\ (NV)\ +\ (\Delta W)\ (CPI)}{(NF)\ (FC\ +\ GC)}$$

- Chart B adjusts the mean-time-between failure (MTBF) for the redundant unit by the appropriate mission duty cycle and phased environmental stress factor, KT. This factor is the sum of the products of the time (T) that the equipment is in operation during each flight phase and the appropriate K factor which is dependent upon the environmental stress to which the equipment is subjected. Some preliminary estimates of mission K factors and times are given in the table below:

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was FO^2/FS. It will be shown that a savings of $446,500 results in the expected program cost by adding the additional redundant unit when the delay costs are considered.

The inertial measurement unit is an outgrowth of the Carousel IV B system presently in use on the Boeing 747 airplanes. It is planned for use with the Carousel VB system presently under development. The unit is a four gimballed platform containing three orthogonal gyros and accelerometers. Servo amplifiers drive servo motor gimbals and receive their error signal from the gyro and accelerometers. The computer which comes with the Carousel V system is not considered part of this inertial measurement unit.

The parameter and variable values estimated for the unit of equipment were as follows:

MTBF 3500 hrs

Procurement Cost $118,000

Weight 57 lbs

Number of vehicles in fleet - 3

Cost of delay $32,000

Ground operations cost $4 x 10^6

Flight operations cost 10^6

KT flight 176

KT ground 50

Number of Flights 445

Fail Safe System is two units

Active form of redundancy

Applying the "GO/NO GO" launch criterion, equation (2), we find that in order for it to be economical to launch with a failure the increse in abort probability (ΔR) must be as below.

Phase of Flight		Time of Operation (T) in Hours	Stress Factor (K)	Mission Duty Cycle Factor (KT)
Launch	Boost	0.1	10	1
	Glide	4.0	1	4
Orbital Operation		160	1	160
Deorbit	Re-entry	1.75	10	17.5
	Glide	2.0	1	2
	Landing	0.25	10	2.5
Launch & De-orbit				27
Launch, Orbital Oper., & De-orbit				187

When possible, the MTBF values used in Chart B should be based on data obtained from past flight experience. The resultant of Chart B is the adjusted MTBF to be used in Chart C.

- Chart C is a plot of operational redundancy levels for equipment in either active redundancy or standby redundancy. The broken lines for standby or off-line redundancy were plotted from the Poisson equation while the solid lines for active or parallel redundancy were plotted from the Binominal equation. The adjusted MTBF is the variable on the abscissa while the change in reliability is the variable on the ordinate. Reliability is defined as the probability that the mission will not be aborted, i.e., the system will still be fail-safe. The expression $\dfrac{(CR)\ (NV) + (\Delta W)\ (CPI)}{(NF)\ (FC + GC)}$ which was obtained from Chart F is entered into Chart C on the ordinate. The adjusted MTBF from Chart B is entered into Chart C on the abscissa. If lines are drawn through these points perpendicular to the axes, the intersection of these two lines on Chart C will represent the level of operational redundancy of a particular unit for which the cost of redundancy is
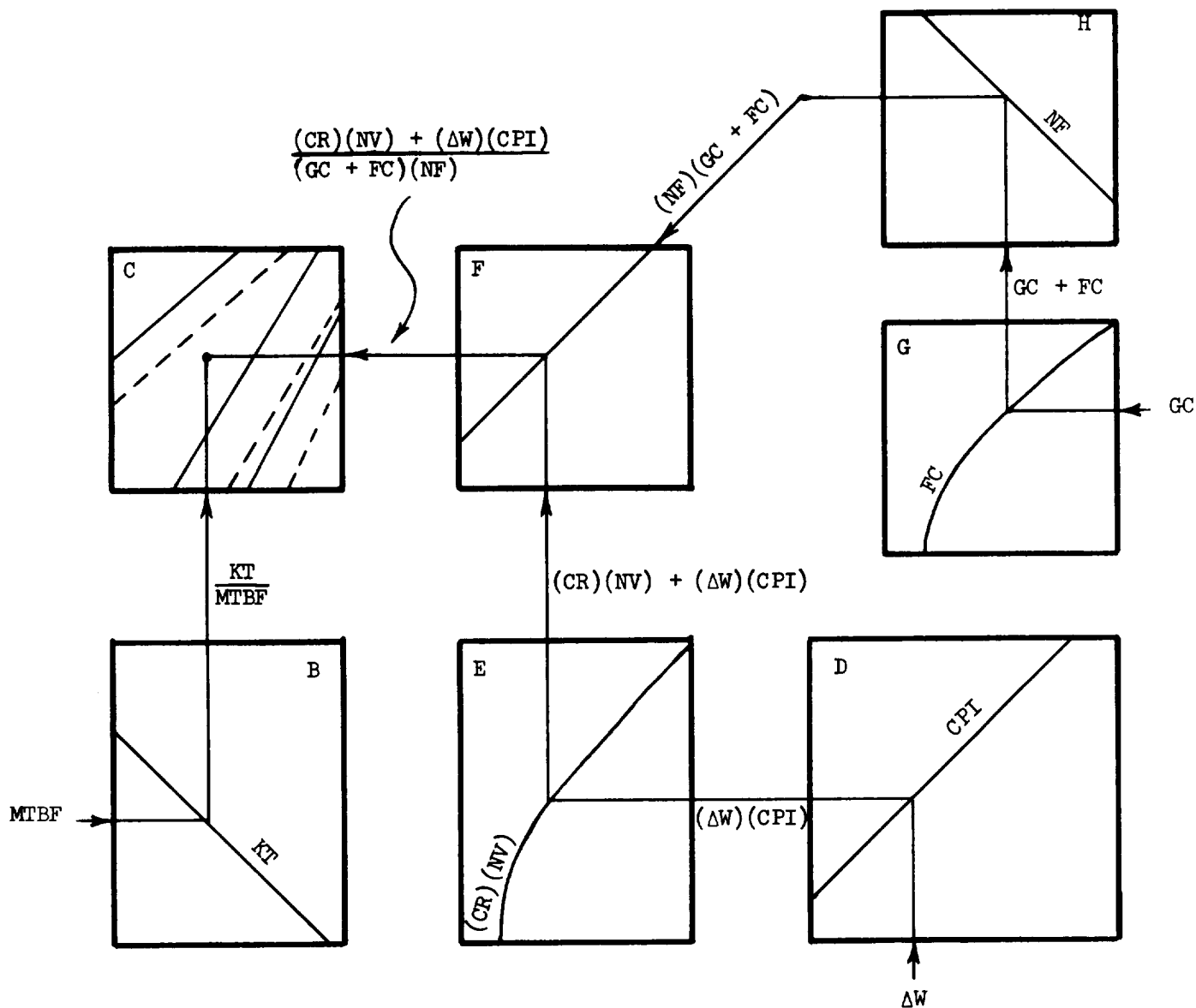
less than the value. If this intersection falls within an operational
redundancy level presently in use or within a redundancy level lower
than the one in use, then the cost of redundancy is greater than the
value and redundancy should not be increased for that particular unit.

(B) Procedure For Use of Nomograph

A simplified nomograph flow is illustrated in Figure I-1. This shows the
required steps of operation to be performed in evaluating the operational re-
dundancy level. A description of each step follows:

- Chart D is entered on the abscissa with the weight of redundancy (ΔW).
  From this point on the abscissa, a vertical line is drawn to intersect
  the appropriate curve of the cost penalty per pound of inert weight
  (CPI). A horizontal line is projected from this intercept into
  Chart E.

- In Chart E the horizontal line from Chart D is extended until it
  intersects the curve which represents the equipment cost per fleet
  (CR)(NV). From this intersection, a line is drawn vertically and
  extended into Chart F.

- Chart G is entered on the ordinate with the operational ground cost
  per mission (GC). From this point, a horizontal line is drawn to
  intersect the proper curve of the operational flight cost per mission
  (FC). From this intersection, a vertical line is drawn and extended
  into Chart H.

- In Chart H the vertical line from Chart G is extended until it inter-
  sects the proper curve for the number of flights in the program (NF).
  From the intersection of these two curves, a horizontal line is drawn
  and extended via transitional grid into Chart F.

FIGURE I-1

ILLUSTRATION OF NOMOGRAPH PROCEDURE

$$\frac{(CR)(NV) + (\Delta W)(CPI)}{(GC + FC)(NF)}$$

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- In Chart F the vertical line from Chart E is extended to intersect the curve which was extended from Chart H into Chart F. The intersection of these two lines is then projected horizontally to the ordinate of Chart F and is the expression $\dfrac{(CR)(NV) + (\Delta W)(CPI)}{(GC + FC)(NF)}$.

- Chart B is entered on the ordinate with the MTBF of the redundant unit. From this point on the ordinate, a horizontal line is drawn to intersect the curve of the mission duty cycle factor (KT) for the redundant unit. At this intersection, a line is drawn vertically upward and extended into Chart C.

- The horizontal and vertical lines from Charts F and B are extended until intersection. The zone in which this intersection occurs specifies the recommended level of operational redundancy for the unit.

APPENDIX II

DESCRIPTION OF "GO/NO GO" LAUNCH CRITERION NOMOGRAPH
AND PROCEDURE FOR ITS USE

(A) Description of Nomograph

The nomograph contained in Figure 6 of Section 5.3 is an updated version of one developed at Grumman for use in in-house shuttle analysis. The function of each chart is as follows:

- Chart A adds the ground cost per mission (GC) to the flight cost per mission (FC) to obtain the total operational cost per mission (GC + FC). This total cost per mission is the wasted cost of an aborted flight. Therefore, all operational ground costs of a mission should be included such as: inspection and post-flight checkout of vehicle, pre-launch checkout, fueling, and launch operations. The flight cost per mission should include post launch operational costs such as: communication costs, tracking costs, Mission Control operational costs, and other flight support costs.

- Chart C multiplies the hours of delay (HD) times the cost per delay hour (CH) to obtain the total cost of the delay $(CH) \cdot (HD)$ which is required to repair the failure.

- Chart B divides the cost of delay $(CH) \cdot (HD)$ by the operational cost per mission (GC + FC) to obtain the ratio $\frac{(CH)(HD)}{(GC + FC)}$. This ratio is the right side of the GO/NO-GO decision inequality: $\Delta R < \frac{(CH) \cdot (HD)}{(GC + FC)}$

- Chart E adjusts the mean-time-between failure (MTBF) for the redundant unit by the appropriate mission duty cycle and phased environmental stress factor, KT. This factor is the sum of the products of the time (T) that the equipment is in operation during each flight phase

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and the appropriate K factor which is dependent upon the environmental stress to which the equipment is subjected. Some preliminary estimates of Mission K factors and times are given in the table below:

Phase of Flight		Time of Operation (T) in Hours	Stress Factor (K)	Mission Duty Cycle Factor (KT)
Launch	Boost	0.1	10	1
	Glide	4.0	1	4
Oribtal Operation		160	1	160
Deorbit	Re-entry	1.75	10	17.5
	Glide	2.0	1	2
	Landing	0.25	10	2.5
Launch & De-orbit				27
Launch, Oribtal Oper., & De-orbit				187

When possible, the MTBF values used in Chart E should be based on data obtained from past flight experience. The resultant of Chart E is the adjusted MTBF to be used in Chart D.

• Chart D shows the amount of operational redundancy which exists after the failure has occurred. The broken lines for standby or off-line redundancy were plotted from the Poisson equation while the solid lines for active or parallel redundancy were plotted from the Binominal equation. The adjusted MTBF is the variable on the abscissa while the change in reliability is the variable on the ordinate. The change in reliability is defined as the increased probability that the mission will be aborted because of the failure.

The expression $\frac{(CH) \cdot (HD)}{(GC + FC)}$ which was obtained from Chart B is entered into Chart D on the ordinate. The adjusted MTBF from Chart E is

entered into Chart D on the abscissa. If lines are drawn through
these points perpendicular to the axes, the intersection of these
two lines on Chart D will represent the level of operational redun-
dancy for which $\Delta R = \dfrac{(CH) \cdot (HD)}{(GC + FC)}$. If this intersection is above and
to the left of the actual redundancy line of the equipment, the cost
of delay will exceed the cost of the added risk and the indicated
decision is to launch. An intersection below and to the right of the
actual redundancy level would show that the cost of added risk is
greater than the cost of delay, indicating that the failure should be
repaired.

(B) Procedure for Use of Nomograph

A simplified nomograph flow is illustrated in Figure II-1. This shows
the required steps of operation to be performed in utilizing the nomograph as
an aid in deciding to launch or to repair the failure. A description of each
step follows:

- Chart A is entered on the abscissa with the operational ground cost
  (GC). From this point on the abscissa, a vertical line is drawn to
  intersect the appropriate curve of the operational flight cost (FC).
  A horizontal line is drawn from this intercept point and extended via
  transitional grid into Chart B.

- Chart C is entered on the ordinate with the hours of delay required
  to repair the failure (HD). From this point, a horizontal line is
  drawn to intersect the proper curve of the cost per hour of delay
  (CH). From this intersection, a vertical line is drawn and extended
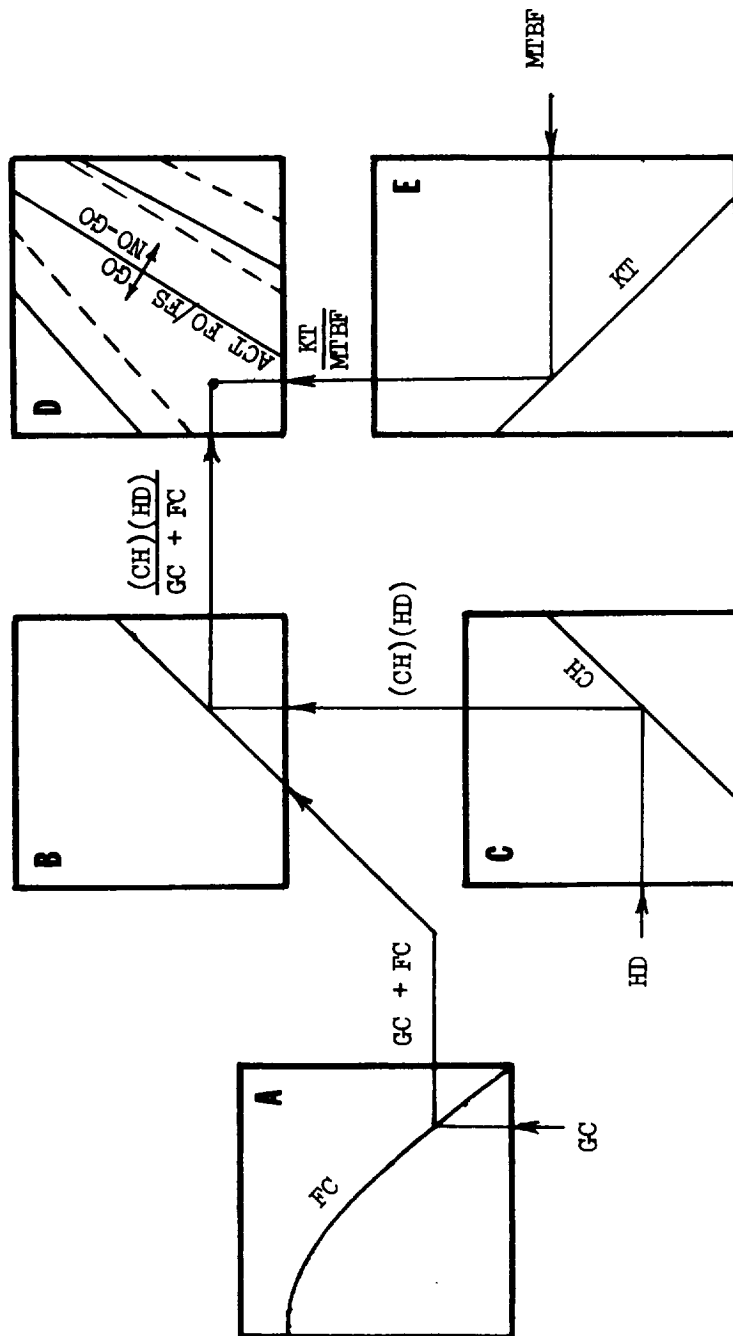 into Chart B.

FIGURE II-1

ILLUSTRATION OF NOMOGRAPH PROCEDURE

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- In Chart B the vertical line from Chart C is extended to intersect the curve which was extended from Chart A into Chart B. This intersection is then projected horizontally to the ordinate of Chart D and is the expression $\frac{(CH) \cdot (HD)}{(GC + FC)}$.

- Chart E is entered on the ordinate with the MTBF of the failed unit. From this point on the ordinate, a horizontal line is drawn to intersect the curve of the mission duty cycle factor (KT) for the failed unit. At this intersection, a line is drawn vertically upward and extended into Chart D.

- The horizontal line from Chart B and the vertical line from Chart E are extended until intersection in Chart D. The zone in which this intersection occurs determines whether it is more economical to launch with the failure or to delay launch and repair the failure.

APPENDIX III

DERIVATION OF INTEGRATED REDUNDANCY
SELECTION CRITERION EQUATIONS

The criterion to be applied in order to ascertain the value of adding the next level of redundancy is dependent upon the launch policy with each amount of redundancy. Therefore, if a failure occurs on the launch pad is it economical to repair it (NO GO) or fly with the failure (GO). This decision changes with each level of redundancy since the change in abort probability (ΔR) changes with additional levels of redundancy. The "GO/NO GO" criterion is to fly without repair when $\Delta R < \dfrac{\text{Cost of Delay}}{\text{Cost of Reflown Mission}}$. Since ΔR eventually decreases as redundancy level increases the launch policy will initially be "NO GO" and then change to "GO." Therefore there are three cases which must be examined:

N-1 Units		N Units
NO GO	→	NO GO
NO GO	→	GO
GO	→	GO

Each case has different costs and benefits principally due to the delay costs and launch states (all up or not). Assumptions which have been made and apply to all three cases examined, include:

- The change in mission abort probability due to addition of redundancy for a single equipment is equal to the change in abort probability for equipment alone.

  Rationale: This is valid for a high reliability objective for mission success. If overall vehicle reliability were .99 then a 1% error in ΔR results from this assumption.

- The operational cost impacts due to additional failures and maintenance action are small relative to the procurement, growth reflown mission and delay costs.

    Rationale: See the sensitivity analysis in Section 6.3 to verify this assumption.

NOTATION USED FOR CRITERION EQUATIONS

The following symbols are used to denote various terms and expressions
for the criterion as follows:

C_1 = Baseline costs with N-1 units (operational, procurement, all
other shuttle vehicle equipment, design, test, etc.)

$P_n(A)$ = Probability of aborting the mission in flight having launched
with n good units

$P_n(F)$ = Probability of a failure during pre-launch checkout with n units
of equipment

C_A = Cost of aborted mission (flight + ground)

N_F = Number of flights in shuttle program

C_R = Procurement cost of a single unit

N_V = Number of vehicles in the fleet

W_R = Weight of a redundant unit

C_P = Cost per pound for vehicle growth

C_D = Cost of delay to repair a malfunction on the launch pad

$E_n(C)$ = Expected program cost with n units of equipment

ΔR_n = Change in abort probability due to addition of the nth unit of
equipment = $P_{n-1}(A) - P_n(A)$

ΔD_n = $P_n(F) - P_{n-1}(F)$ = change in frequency of a delay on the launch
pad due to addition of the nth unit of equipment

CASE I - N-1 Units GO; N Unit GO

The costs and benefits incurred due to the addition of the nth unit are:

<table>
<tr><th>COSTS</th><th>BENEFITS</th></tr>
<tr><td>Procurement</td><td>Reduced abort frequency</td></tr>
<tr><td>Vehicle Growth</td><td></td></tr>
</table>

Additional redundancy is suggested if the expected program cost is lower with n units than with n-1 units.

Expected program cost with n units $(E_n(C))$:

$$C_1 + \left\{ P_n(A) \cdot \left(1 - P_n(F)\right) + P_{n-1}(A) \cdot P_n(F) \right\} \cdot C_A \cdot N_F$$

$$+ (C_R) \cdot (N_V) + (W_R) \cdot (C_P)$$

Expected program cost with n-1 units $(P_{n-1}(C))$

$$C_1 + \left\{ P_{n-1}(A) \cdot \left(1 - P_{n-1}(F)\right) + P_{n-2}(A) \cdot P_{n-1}(F) \right\} C_A \cdot N_F$$

$\left\{ \ \right\}$ = The bracket term is the expected probability of aborting the mission and is the average based on the various equipments operating at launch.

Add redundancy if expected program cost decreases.

$$E_N(C) < E_{N-1}(C)$$

$$C_1 + \left\{ P_n(A) \cdot \left(1 - P_n(F)\right) + P_{n-1}(A) \cdot P_n(F) \right\} C_A N_F$$

$$+ C_R \cdot N_V + W_R \cdot C_P < C_1 + \left\{ P_{n-1}(A) \cdot \left(1 - P_{n-1}(F)\right) + P_{n-2}(A) \cdot P_{n-1}(F) \right\} \cdot C_A N_F$$

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$$P_n(A) - P_n(A) \cdot P_n(F) + P_{n-1}(A) \cdot P_n(F) - P_{n-1}(A) + P_{n-1}(A) \cdot P_{n-1}(F)$$

$$- P_{n-2}(A) \cdot P_{n-1}(F) < - \frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F}$$

let $\Delta R_n = -P_n(A) + P_{n-1}(A)$

then,

$$- \Delta R_n + (\Delta R_n) \cdot P_n - P_{n-1}(F) \cdot \Delta R_{n-1} < - \frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F}$$

or

$$\Delta R_n \left(1 - P_n(F)\right) > \frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F} - P_{n-1}(F) \, \Delta R_{n-1}$$

$$\Delta R_n > \left\{ \frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F} - P_{n-1}(F) \cdot \Delta R_{n-1} \right\} \cdot \frac{1}{1 - P_n(F)}$$

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CASE II - n-1 Units NO GO; n Units GO

The cost and benefits incurred due to addition of the nth unit are:

Costs	Benefits
Procurement	Reduced abort frequency
Vehicle Growth	Eliminates delays on launch pad if a failure occurs

Expected program cost with n units ($E_n(C)$):

Baseline cost + reflown mission costs + procurement costs + growth costs.

$$C_1 + \left\{ P_N(A) \cdot \left(1 - P_n(F)\right) + P_{n-1}(A) \cdot P_n(F) \right\} C_A \cdot N_F$$

$$+ (C_R) \cdot (N_V) + (W_R) \cdot (C_P)$$

Expected program cost with n-1 units ($E_{n-1}(C)$):

Baseline cost + reflown mission costs + delay costs

$$C_1 + P_{n-1}(A) \cdot C_A \cdot N_F + P_{n-1}(F) \cdot C_D \cdot N_F$$

Add redundancy if

$$E_n(C) < E_{n-1}(C)$$

$$C_1 + \left\{ P_N(A) \cdot \left(1 - P_n(F)\right) + P_{n-1}(A) \cdot P_n(F) \right\} \cdot C_A \cdot N_F$$

$$+ C_R \cdot N_V + W_R \cdot C_P < C_1 + P_{n-1}(A) \cdot C_A \cdot N_F + P_{n-1}(F) \cdot C_D \cdot N_F$$

or:

$$P_N(A) - P_N(A) \cdot P_n(F) + P_{n-1}(A) \cdot P_n(F) - P_{n-1}(A)$$

$$< - \frac{(C_R \cdot N_V + W_R \cdot C_P)}{C_A \cdot N_F} + P_{n-1}(F) \cdot \frac{C_D}{C_A}$$

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let $\Delta R_N = - P_N(A) + P_{N-1}(A)$

then:

$$-\Delta R_N + (\Delta R_N) \cdot (P_n(F)) \cdot < - \frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F} + P_{n-1}(F) \cdot \frac{C_D}{C_A}$$

$$\Delta R_n > \frac{1}{(C_A) \cdot (1-P_n(F))} \cdot \left\{ \frac{C_R \cdot N_V + W_R \cdot C_P}{N_F} - (C_D) \cdot P_{n-1}(F) \right\}$$

CASE III - n-1 Units: "NO GO"; n Units "NO GO"

The costs and benefits incurred due to addition of the nth unit are:

Costs	Benefits
Procurement	Reduced abort frequency
Vehicle Growth	
Additional delays	

Expected program cost with n units of equipment = $E_n(C)$.

$E_n(C)$ = Baseline cost + reflown mission costs + procurement costs + growth costs + delay costs.

$$= C_1 + P_N(A) \cdot C_A \cdot N_F + (C_R) \cdot (N_V) + (W_R) \cdot (C_P)$$

$$+ P_n(F) \cdot C_D \cdot N_F$$

Expected program cost with n-1 units of equipment = $E_{n-1}(C)$.

$E_{n-1}(C)$ = Baseline cost + reflown mission costs + delay costs

$$= C_1 + P_{n-1}(A) \cdot C_A \cdot N_F + P_{n-1}(F) \cdot C_D \cdot N_F$$

Add the nth unit if the expected program cost is lower than with n-1 units of equipment.

$$E_n(C) < E_{n-1}(C)$$

$$C_1 + P_N(A) \cdot C_A \cdot N_F + (C_R) \cdot (N_V) + (W_R) \cdot (C_P) + P_n(F) \cdot C_D \cdot N_F < C_1$$

$$+ P_{n-1}(A) \cdot C_A \cdot N_F + P_{n-1}(F) \cdot C_D \cdot N_F$$

or

$$P_N(A) - P_{N-1}(A) < -\frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F} + \frac{C_D \cdot N_F}{C_A \cdot N_F} \left\{ P_{N-1}(F) - P_n(F) \right\}$$

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Let $\Delta R_n = P_{N-1}(A) - P_N(A)$

$\Delta D_n = P_n(F) - P_{n-1}(F)$

Then

$$\Delta R_n > \frac{C_R \cdot N_V + W_R \cdot C_P}{C_A \cdot N_F} + \frac{C_D}{C_A} \cdot \Delta D_n$$